Self-crossing boundary Lefschetz fibrations

Aldo Witte

(KU Leuven)

November 17, 2021
16:00-17:00h in M.G.004
Analysis & Geometry Seminar, Antwerpen

A common theme in differential geometry is the interplay between specific geometric structures and certain types of maps. A simple example is when the existence of a (singular) fibration, together with a geometric structure on the base and the fibre induced a geometric structure on the total space. A concrete example comes from symplectic geometry: Here Lefschetz fibrations are used to establish broad existence results. Another area from which singular fibrations arise is from non-principal torus actions. Even though these actions are not free, when a torus with half the dimension of the manifold acts on it, under reasonable assumptions, the quotient will be a manifold with boundary. The coupling of torus actions and geometric structures leads to many fruitful concepts, the highlight being toric geometry. The Lefschetz and toric pictures come together in semi-toric geometry, where the maps under consideration are allowed to have both singularities. We aim to decouple these maps from the geometric structure, and study them on their own: We call them (self-crossing) boundary Lefschetz fibrations.

We will show that these are also precisely the maps used to study an, a priori seemingly unrelated, geometric structure: generalized complex structures. We will prove abstract results for boundary Lefschetz fibrations, in particular a connected sum procedure and a nodal trade procedure a la Symington, and use them to construct examples of generalized complex structures.

This is a joint work with Gil Cavalcanti and Ralph Klaasse.