Miniworkshop “From toric to hyperbolic”
University of Antwerp, Belgium
March 15-16, 2022

Hyperbolic singularities in the presence of S^1-actions and Hamiltonian PDEs

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Hamiltonian systems are dynamical systems which have at least one conservation law. These systems are of particular interest because they allow us to use geometric tools to obtain dynamical results. This thesis focuses on two distinct types of Hamiltonian systems: proper S^1-systems and Hamiltonian PDEs.

Proper S^1-systems are Hamiltonian systems on four dimensional manifolds, where we have two conservation laws one of which is a proper map inducing an S^1-action. The presence of the S^1-action allows us to link the minimal period (dynamical feature) of that S^1-action to the local shape of hyperbolic fibers (topological feature). This allows us to establish a one-to-one correspondence between the topology of hyperbolic fibers and a graph theoretical construction, called a generalized bouquet. After the theoretical classification of hyperbolic fibers we focus on explicit examples. We study the bifurcation behaviour of a family of proper S^1-systems and discuss what happens locally around hyperbolic fibers.

For the investigation of Hamiltonian PDEs, we start with a ‘triholomorphic’ Dirac-type equation, called the Cauchy-Riemann-Fueter equation, on a so-called hyperkähler manifold that can be transformed into a Hamiltonian PDE. Then, we discuss scale manifolds as preferred underlying function space for the study this equation. Finally, we describe the problems concerning convergence behaviour.