Convergence Analysis of Non-Euclidean Methods for Nonsmooth Nonconvex Optimization

Masoud Ahookhosh

(University of Antwerp)

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Convergence analysis of methods for minimizing problems with nonsmooth and nonconvex objective functions is a challenging task, which requires tools from the variational and nonsmooth analysis. Most of such objectives are the sum of two or more functions that some of them are smooth and the others are nonsmooth. In the classical setting, the smooth part of the objective is assumed to have Lipschitz continuous gradients, while there are abundant practical problems that do not comply with this assumption (e.g., polynomial with degree more than four), which leads to a major difficulty for handling such optimization problems.

One of the main tools for designing optimization methodologies are proximal point operators (in the Euclidean setting) that are very useful for structured (non)smooth (non)convex optimization problems; however, they cannot be used for problems with non-Lipschitz gradients. In this talk, we introduce the relative smoothness condition and modify the proximal operators using Bregman distances, which lead to non-Euclidean tools to deal with problems with non-Lipschitz gradients. Next, we introduce Bregman Moreau and forward-backward envelope and investigate their nice differential properties under some mild assumptions (e.g., prox-regularity and twice epi-differentiability). Then, we can use these tools to design some method (called BELLA) combing the non-Euclidean forward-backward method with Newton-type approaches. Finally, we will show the subsequential, global, and superlinear convergence of this method to a non-isolated critical point of such problems for functions satisfying the celebrated Kurdyka-Lojasiewicz property.