7th international conference on
Finite Dimensional Integrable Systems in Geometry and Mathematical Physics

FDIS 2023

University of Antwerp, 7-11 August 2023

Organization committee:

Misha Bialy (Tel Aviv), Sonja Hohloch (Antwerp), Vladimir Matveev (Jena), Sergei Tabachnikov (Penn State)

Local organization committee:

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Misha Bialy (Tel Aviv), Alexey Bolsinov (Loughborough), Allan Fordy (Leeds), Boris Kruglikov (Tromsø), Andrzej Maciejewski (Zielona Gora), Álvaro Pelayo (Madrid), Vladimir Rubtsov (Angers), Dmitry Treschchev (Moscow)

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1 Practical information

1.1 Website of the conference

https://www.uantwerpen.be/fdis2023/

1.2 Wifi

You can use your eduroam account within the university buildings.

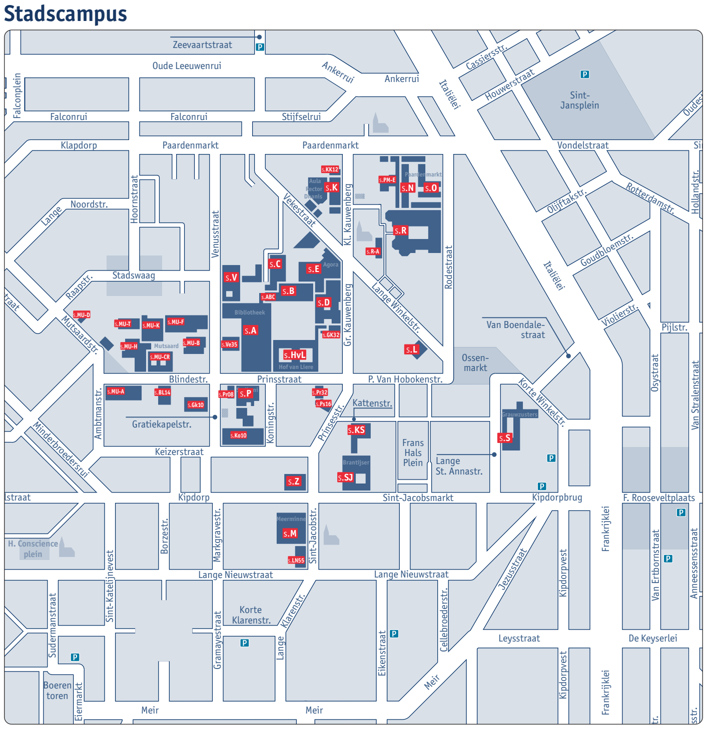
1.3 Emergency numbers in Belgium

Fire and Ambulance: **112**
Police: **101**

1.4 Stadscampus map

More information is on the website:
https://www.uantwerpen.be/en/about-uantwerp/campuses/stadscampus/

Stadscampus



s.**A** - Library, Prinsstraat 13
s.**B** - Prinsstraat 13
s.**BL14** - **Open University / Centrum West** - Blindestraat 14
s.**C** - Prinsstraat 13
s.**D** - Grote Kauwenberg 18
s.**E** - **Agora/Sports centre** - Grote Kauwenberg 2
s.**Gk10** - **International Relations Office** - Gratiekapelstraat 10
s.**GK32** - **Ruusbroec association** - Grote Kauwenberg 32-34
s.**HvL** - **Hof van Liere** - Prinsstraat 13
s.**HvL** - **University Club** - Prinsstraat 13b
s.**K** - **Aula Rector Dhanis** - Kleine Kauwenberg 14
s.**KK12** - **Linguapolis** - Kleine Kauwenberg 12
s.**Ko10** - **komida / Labotheek** - Koningstraat 8
s.**KS** - **Het Brantijser** - Kattenstraat 10
s.**L** - Lange Winkelstraat 40-42
s.**M** - **De Meerminne** - Sint-Jacobstraat 2
s.**MU** - **Mutsaard** - Mutsaardstraat 29
s.**MU-A** - **Mutsaard** - Ambtmanstraat 1
s.**N** - **De Paardenmarkt** - Paardenmarkt 94
s.**O** - **De Paardenmarkt** - Paardenmarkt 94
s.**P** - **Scribani** - Prinsstraat 10
s.**Pr08** - **Linguapolis** - Prinsstraat 8
s.**Pr32** - **Zomaar een dak** - Prinsstraat 32
s.**PM** - Paardenmarkt 92
s.**Ps16** - **Universitas - Rubi - Student Council** - Prinsesstraat 16
s.**R** - Rodestraat 14
s.**R-A** - **Annexe** - Lange Winkelstraat 9
s.**S** - **Grauwzusters** - Lange Sint-Annastraat 7
s.**SJ** - **Het Brantijser** - Sint-Jacobsmarkt 13
s.**V** - Venusstraat 23
s.**Ve35** - **Moral consultant / Prevention Service** - Venusstraat 35
s.**Z** - **Peter Benoit** - Kipdorp 61

University of Antwerp

1.5 Lecture halls

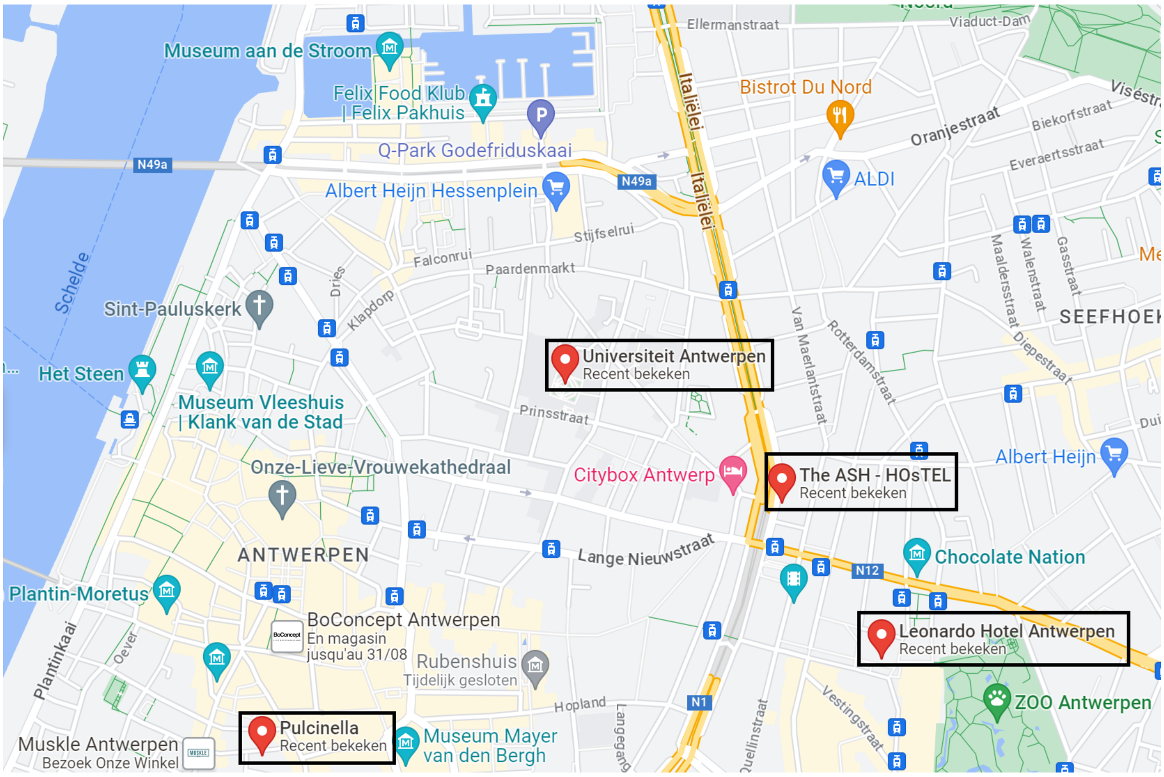
All conference activities will take place in **Building C of the Stadscampus of the University of Antwerp** (indicated in the map in Section 1.6).

All plenary lectures will take place in lecture hall S.C.003. The parallel lectures will take place in lecture halls S.C.002 and S.C.003. We also have access to room S.C.001 for discussions and collaborations. The poster session will take place in the corridor leading to rooms S.C.001, S.C.002, and S.C.003.

Building C will open around 08:00, and will close around 21:00. **However, after 17:00 it will no longer be possible to enter Building C** without an access card (although it will still be possible to exit).

1.6 Map of the city

The location of the "Stadscampus" of the University of Antwerp, the hotel Leonardo, and the two hostels are shown in the map below.



1.7 Getting around Antwerp

- The main train station, the hotel, the hostels, and the conference hall are all within walking distance, and there are trains and buses.
- You can also make an account to rent bikes from stands throughout the city, using this website: https://www.velo-antwerpen.be/
- Further information about getting around the city can be found at the website https://www.slimnaarantwerpen.be/

1.8 How to reach lecture hall C.003

To enter the lecture hall, first navigate to Grote Kauwenberg 31, then follow the steps described below:

Step 0

Step 1

Step 2

Step 3

Step 4

Step 5

Step 6

Step 7

2 Schedule

The yellow talks (and Tsodikovich's talk) are in room S.C.003. The green talks are parallel sessions. Each parallel talk is 30 minutes long. The talk listed on the left will be in room S.C.003 and the talk listed on the right will be in room S.C.002.

	Monday	Tuesday	Wednesday	Thursday	Friday
9:00 - 9:30	Bialy (9:00 - 9:50)	Bolsinov (9:00 - 9:50)	Hone (9:00 - 9:50)	Bor (9:00 - 9:50)	Zambon (9:00 - 9:50)
9:30 - 10:00					
10:00 - 10:30	Coffee and Registration	Coffee break	Coffee break	Coffee break	Coffee break
10:30 - 11:00	Van Moerbeke (10:30 - 11:20)	Poster session	Bangert (10:30 - 11:20)	Dragovic (10:30 - 11:20)	Fehér / Los
11:00 - 11:30					Dearricott / Anand Singh
11:30 - 12:00	Jovanović (11:30-12:10)	Efstathiou (11:30 - 12:10)	Montgomery (11:30 - 12:10)	Izosimov (11:30 - 12:10)	Mironov (11:50 - 12:30)
12:00 - 12:30					
12:30 - 13:00	Lunch break (12:10-14:30)	Lunch break (12:10-14:30)		Lunch break (12:10-14:30)	Lunch break (12:30-14:30)
13:00 - 13:30					
13:30 - 14:00					
14:00 - 14:30					
14:30 - 15:00	Vizman (14:30 - 15:20)	Tolman (14:30 - 15:20)		Arnold (14:30 -15:20)	Suris (14:30 - 15:20)
15:00 - 15:30					
15:30 - 16:00	Vollmer	Palmer / Marquette		Tsodikovich	Closing (15:30 - 15:40)
16:00 - 16:30	Coffee Break	Coffee break		Coffee break	
16:30 - 17:00	Witte / Marciniak	Akpan / Kim		Khimshiashvili / Kaplan	
17:00 - 17:30	Reception (17:00 - 19:00)	Martynchuk (17:10 - 18:00)		Calini (17:10 - 18:00)	
17:30 - 18:00					
18:00 - 18:30					
18:30 - 19:00					

3 Abstracts Monday 7th

Misha Bialy (Tel Aviv University)

Self-Bäcklund curves via Lame equation: explicit construction and applications
In my talk I shall discuss a recent paper joint with Gil Bor and Sergei Tabachnikov on self-Bäcklund centroaffine curves. We describe general properties of these curves and provide an explicit description of them in terms of elliptic functions. Our work is a centroaffine counterpart to the study done by F. Wegner of a similar problem in Euclidean geometry, related to Ulam's floating problem of describing the (2-dimensional) bodies that float in equilibrium in all positions and to bicycle kinematics. Joint with Gil Bor and Sergei Tabachnikov.

Pierre Van Moerbeke (UC Louvain)

The surprising mathematics of tiling models
In this lecture I will discuss random coverings of large geometrical domains (convex and non-convex) with dominos or lozenges. For large size domains these "tilings" will typically lead to different regions within the domain, separated by algebraic curves ; these regions can be solid, liquid or gas-like. The fluctuations of the tiles near those curves and their singularities obey universal probability laws.

Božidar Jovanović (Mathematical Institute SANU, Belgrade)

Almost multiplicity free subgroups and polynomial integrability of sub-Riemannian geodesic flows
We classify almost multiplicity free subgroups K of compact simple Lie groups G. The problem is related to the integrability of Riemannian and sub-Riemannian geodesic flows of left-invariant metric defined by a specific extension of integrable systems from T^*K to T^*G.

Cornelia Vizman (West University of Timisoara)

Dynamics of rotationally symmetric vortex sheets
We describe the coadjoint orbits of the group of volume preserving diffeomorphisms of R3, associated to the motion of vortex sheets in ideal 3D fluids. These are nonlinear Grassmannians of compact surfaces in R3, enclosing a fixed volume and endowed with a closed 1-form describing the vorticity density. These coadjoint orbits are shown to be prequantizable if the period group of the 1-form and the volume enclosed by the surface satisfy an Onsager-Feynman relation. We study the Hamilton equations on coadjoint orbits of vortex sheets for Khesin's Hamiltonian function. If the vorticity density has a discrete period group and is non-vanishing, then the vortex sheet is given by a surface of genus one fibered by its vortex lines over a circle. We focus on such vortex sheets with rotational symmetries and with vortex lines the parallel circles.

Andreas Vollmer (University of Hamburg)

Conformal geometry and non-degenerate second order superintegrable systems
Second-order superintegrable systems play an important role in mathematics and physics, and their classification is an ongoing problem (to date only achieved in low dimension). We have developed a framework (joint with J. Kress and K. Schöbel) that is manageable in arbitrary dimension, encoding a superintegrable system via a (0,3)-tensor field (called structure tensor).

The focus of the talk is going to be on conformal superintegrability, which is preserved under conformal rescalings of the system. We determine concise algebraic integrability conditions for a large class of systems (both conformal and proper ones), which lead to previously unknown obstructions in dimension 4 and higher. Particularities in dimension 2 will also be addressed.

Non-degenerate second-order superintegrable systems on constant curvature spaces are the best understood examples: their structure tensor can be reconstructed from the knowledge of two scalar functions (a property shared with a wider class of systems). Their conformal scale functions are characterised via eigenfunctions of the Laplace operator. Alternatively, a geometric characterisation via the flatness of a certain torsion-free connection exists (and has interrelations to affine hypersurfaces).

Aldo Witte (University of Antwerp)

Toric symplectic structures with elliptic singularities
Singular symplectic geometry studies Poisson structures with mild singularities. Since the introduction of log symplectic structures by Guillemin-Miranda-Pires it has turned into a vibrant area of Poisson geometry. They can be used to model classical mechanical systems such as the restricted three-body problem. In this talk we will introduce another class of singular symplectic structures, which surprisingly enough find their origin in the completely different world of string theory. These are the so-called elliptic toric structures, and are closely related to ordinary toric symplectic structures. We will introduce certain Lie algebroids which can be used to study them, and establish a classification a la Delzant. Essential in this story is the study of connections for non-principal torus actions using Lie algebroids, which also has applications to smooth symplectic geometry.

Krzysztof Marciniak (Linköping University)

Coupled KdV stationary systems and their Stäckel representations
In this talk I revisit the concept of stationary cKdV system and prove that every N-field stationary cKdV system can be written, after a careful reparametrization of jet variables, as a corresponding classical separable Stäckel system in N+1 different ways. For each of these N+1 parametrizations I will present an explicit map between the jet variables and the separation variables of the system. Each pair of Stäckel representations of the same stationary cKdV system, when considered in the phase space extended by Casimir variables, is connected by an appropriate Miura map, which leads to an $(N+1)$-Hamiltonian formulation for the considered N-field stationary cKdV system.

4 Abstracts Tuesday 8th

Alexey Bolsinov (Loughborough University)

Nijenhuis Geometry: conservation laws, symmetries and applications to geodesically equivalent metrics.
This talk continues Nijenhuis Geometry research program started few years ago (joint project with A.Konyaev and V.Matveev). The main object of study within this program are $(1,1)$-tensor fields with vanishing Nijenhuis torison known as Nijenhuis operators. They pop up in many, a priori unrelated, branches of mathematics, so it makes sense to develop a general

theory of Nijenhuis operators and then to apply the results and methods obtained wherever these operators appear (e.g., in the theory of geodesically equivalent metrics as in the present talk). This approach treats a Nijenhuis operator as a primary object, even if it initially appeared as a secondary object in the study of another structure.

In this talk, the partner structure for a Nijenhuis operator L is a metric g which is geodesically compatible to it. We describe all of such metrics for a gl-regular Nijenhuis operator L. It appears that the answer is naturally given in terms of strong symmetries of L. Next, we show that a certain evolutionary PDE system of hydrodynamic type constructed from L preserves the property of a curve to be a g-geodesic. Moreover, the restriction (reduction) of this system onto the set of g-geodesics is naturally equivalent to the Poisson action of $\mathbb{R}^n$ on the cotangent bundle generated by the integrals coming from geodesic compatibility.

Konstantinos Efstathiou (Duke Kunshan University)

Rotation 1-forms and non-compact monodromy from topological gluing
We consider the notion of rotation 1-forms and how they can be used to compute the monodromy of a compact integrable Hamiltonian fibration. Then we introduce non-compact monodromy, that is, monodromy for integrable Hamiltonian systems with non-compact fibers. This is done by compactifying the fibration through an appropriate topological gluing and considering the monodromy of the resulting compact fibration. We discuss the relation between rotation 1-forms and topological gluing and we extend these results to fractional monodromy.

Susan Tolman (UIUC)

Complexity one nondegenerate integrable systems
Let (M, ω) be a compact, connnected $2n$-dimensional symplectic manifold. Fix $f : M \to \mathbb{R}^n$ such that (M, ω, f) is an integrable system. If M is four-dimensional then we say that (M, ω, f) is semitoric if (1) every singular point is non-degenerate without hyperbolic blocks, and (2) the first component of f generates a circle action. Semitoric systems have been extensively studied and have many nice properties: for example, the preimages $f^{-1}(x)$ are all connected. Unfortunately, although there are many interesting examples of semitoric systems, the class has some limitation. For example, there are blowups of $S^2 \times S^2$ with Hamiltonian circle actions which cannot be extended to semitoric systems. We expand the class of semitoric systems in two ways. First, we allow symplectic manifolds of arbitrary dimension, as long as the first $n-1$ coordinates of generate an $n-1$-dimensional "complexity one" torus action. Second, we allow all non-degenerate singularities, including those with hyperbolic block. Although, in this larger class, it's no longer true that the preimage $f^{-1}(x)$ is always connected, D. Sepe and I can prove, For example, if each preimage contains at most one singular point then the preimages are all connected iff no singular point with a connected torus stabilizer has contains a hyperbolic block. I will also discuss work in progress with J. Palmer, where we study which complexity one torus actions can be extended to nondegenerate integrable systems.

Joseph Palmer (University of Antwerp and UIUC)

Families of four-dimensional integrable systems with S^1-symmetries
In this talk I will explain some new techniques for understanding and constructing integrable systems in dimension four with S^1-symmetries. The main idea is to consider a family of integrable systems defined on a fixed symplectic manifold with a fixed Hamiltonian S^1-action.

By varying the Hamiltonian which does not generate the S^1-action, we can induce certain bifurcations in the system, such as Hamiltonian-Hopf bifurcations (including nodal trades). In this way, we are able to start with a well-understood system, such as a toric system, and deform it into a more complicated system, such as semitoric or hypersemitoric system. These techniques allow us to obtain, in particular, explicit examples of semitoric systems which are important in the semitoric minimal models program. The underlying S^1-space for these systems includes certain features (such as $\mathbb{Z}_k$-spheres) whose presence in integrable systems was difficult to deal with using pre-existing techniques. This work is joint with Yohann Le Floch.

Ian Marquette (The University of Queensland)

Algebraic constructions of superintegrable systems from commutant

It was discovered how polynomial algebras appear naturally as symmetry algebra of quantum superintegrable quantum systems. They provide insight into their degenerate spectrum, in particular for models involving Painlevé transcendents for which usual approaches of solving ODEs and PDEs cannot be applied. Those algebraic structures extend the scope of usual symmetries in context of quantum systems, but they also been connected to different areas of mathematics such as orthogonal polynomials. Among them, the well-known Racah algebra which also admit various generalisations.

I will take a different perspective on those algebraic structures which is based on Lie algebras, their related enveloping algebras, partial Casimir and commutant. The talk will present various explicit examples, and in particular the symmetry algebra of the generic superintegrable systems on the 2-sphere and 3-sphere which can be understood in a purely algebraic manner using an underlying Lie algebra

Dinmukhammed Akpan (Lomonosov Moscow State University)

Nijenhuis operators of small coranks and their applications.

We will study Nijenhuis operators in arbitrary dimensions when corank of Jacobi matrix of invariants equals to one and their applications in the theory of projectively equivalent metrics in small-dimensional cases.

Wookyung Kim (Lancaster University)

Integrable deformation of cluster map

An integrable deformation of a cluster map is an integrable Poisson map which is composed of a sequence of deformed cluster mutations, namely, parametric birational maps preserving the presymplectic form but destroying the Laurent property, which is a necessary part of the structure of a cluster algebra. However, this does not imply that the deformed map does not arise from a cluster map: one can use so-called Laurentification, which is a lifting of the map into a higher-dimensional space where the Laurent property is recovered, and thus the deformed map can be generated from elements in a cluster algebra. This deformation theory was introduced recently by Hone and Kouloukas, who presented several examples, including deformed integrable cluster maps associated to Dynkin types A_2, A_3 and A_4. In this talk, we will consider the deformation of integrable cluster map corresponding to the general even dimensional case, Dynkin type A_{2N}.

Nikolay Martynchuk (University of Groningen)

New topological invariants of QFDIS
A popular research direction within the field of (quantum) integrable systems is the topological theory that started with the classical works of Duistermaat, Cushman and Vu Ngoc on classical and quantum monodromy and Fomenko and Zieschang on their marked molecules. We show how a mild generalisation of these invariants can allow one to study systems where this theory has not been fully applied before, such e.g. the A2 singularity, geodesics flows on non-compact manifolds and certain particle systems. Time-permitting, a connection to symplectic geometry will also be discussed.

The talk is based on joint works with K. Efstathiou, H. Waalkens, H.R. Dullin, and E.A. Kudryavtseva.

5 Abstracts Wednesday 9th

Andrew Hone (University of Kent)

Deformed cluster mutations and discrete integrability
We consider deformations of sequences of cluster mutations in finite type cluster algebras, which destroy the Laurent property but preserve the presymplectic structure defined by the exchange matrix. The simplest example is the Lyness 5-cycle, arising from the cluster algebra of type A_2 : this deforms to the Lyness family of integrable symplectic maps in the plane. For types A_3 and A_4 , we find suitable conditions such that the deformation produces a two-parameter family of Liouville integrable maps (in dimensions two and four, respectively). Deformations of Zamolodhchikov periodicity for other root systems will briefly be surveyed. We also perform Laurentification for these maps, by lifting them to a higher-dimensional space of tau functions with a cluster algebra structure, where the Laurent property is restored.

Victor Bangert (Universität Freiburg)

Encounters with Integrable Systems
I will describe some encounters of a geometer with integrable systems. One of them was successful and led to a result on isometry-invariant geodesics. The others were fruitless attempts on questions that remain open problems (some of which well-known).

Richard Montgomery (UC Santa Cruz)

Some integrable subRiemannian geodesic flows
What are the completely integrable subRiemannian geometries "over" the Euclidean plane? A few weeks ago the latest in a list of such geometries was uncovered arxiv.org/abs/2307.13839. We describe some of the geometries in this list and their geodesics. (See arxiv.org/abs/2109.13835 arxiv.org/abs/2010.04201, and arxiv.org/abs/1103.2818 .) We suggest another 'tower' of geometries over the plane to pursue towards integrability.

6 Abstracts Thursday 10th

Gil Bor (CIMAT, Guanajuato, Mexico)

Bicycling geodesics in 3D are Kirchhoff rods

A bicycle path in $\mathbb{R}^n$ is a pair of parametrized curves, the "front" and "back" tracks, traced out by the endpoints of a moving line segment of fixed length which is tangent to the back track at every moment. Bicycle geodesics are bicycle paths whose front track's length is critical among all bicycle paths connecting two given placements of the line segment. For general n the problem reduces to the case of n=3, in which case the front tracks of these geodesics turn out to be a subclass of a family of curves introduced in 1859 by G. Kirchhoff, generalizing the planar elastic curves of J. Bernoulli and L. Euler. A conceptual explanation is still missing.

Vladimir Dragovic (The University of Texas at Dallas)

Bridging Statistics with Geometry and Mechanics

We emphasize the importance of bridges between statistics, mechanics, and geometry. We develop and employ links between pencils of quadrics, moments of inertia, and linear and orthogonal regressions. For a given system of points in R^k representing a sample of a full rank, we construct a pencil of confocal quadrics which appears to be a useful geometric tool to study the data. Some of the obtained results can be seen as generalizations of classical results of Pearson on orthogonal regression. Applications include statistics of errors-in-variables models (EIV) and restricted regressions, both ordinary and orthogonal ones. For the latter, a new formula for test statistic is derived, using the Jacobi elliptic coordinates associated to the pencil of confocal quadrics. The developed methods and results are illustrated in natural statistics examples. The talk is based on a joint work with Borislav Gajić.

Anton Izosimov (University of Arizona)

Planar networks and simple Lie groups

Elements of the matrix group GL(n) can be encoded by weighted planar networks (graphs with numbers written on edges/faces). Such graphical representation is useful for studying matrix factorizations, totally positive matrices etc. Furthermore, Gekhtman, Shapiro, and Vainshtein showed that such networks also capture Poisson geometry of GL(n) endowed with its standard multiplicative Poisson bracket. In the talk I will present similar graphical representations of simple Lie groups of types B and C. Time permitted, I will also talk about an extension of this to loop groups.

Maxim Arnold (The University of Texas at Dallas)

Ideal polygon foldings

Folding of the ideal polygon an its j-th vertex reflects the vertex in the corresponding short diagonal. We will show that compositions of such foldings provide Liouville integrable system on the moduli space of ideal polygons. This is joint work with Anton Izosimov.

Daniel Tsodikovich (Tel Aviv University)

Billiard with rotational symmetry
We generalize the following simple geometric fact: the only centrally symmetric convex curve of constant width is a circle. Billiard interpretation of the condition of constant width reads: a planar curve has constant width, if and only if, the Birkhoff billiard map inside the planar curve has a rotational invariant curve of 2-periodic orbits. We generalize this statement to curves that are invariant under a rotation by angle $\frac{2\pi}{k}$, for which the billiard map has a rotational invariant curve of k-periodic orbits. Similar result holds true also for Outer billiards and Symplectic billiards. Finally, we consider Minkowski billiards inside a unit disc of Minkowski (not necessarily symmetric) norm which is invariant under a linear map of order $k \geq 3$. We find a criterion for the existence of an invariant curve of k-periodic orbits. As an application, we get rigidity results for all those billiards. Joint work with Misha Bialy.

Giorgi Khimshiashvili (Ilia State University)

Topological aspects of generalized Sklyanin algebras
We describe an algebraic method for obtaining topological information on the fibers of moment mapping of generalized Sklyanin algebras introduced by G.Khimshiashvili and studied in some detail by R.Przybysz. In the case of classical Sklyanin algebras, a complete description of the topology and bifurcations of the fibers of moment mapping will be obtained using the algebraic formulae for the Euler characteristic and mapping degree.

Melike Kaplan (Kastamonu University)

The Auto Backlund Transformations of Nonlinear Partial Differential Equations
In this study, we constructed Auto-Backlund and Cole-Hopf transformations for a model. We found numerous soliton-like solutions represented by the hyperbolic, trigonometric, and exponential function waves. Moreover, we reported the dynamics of lump, lump-kink, breather, two-wave and three-wave solutions using the Hirota bilinear technique.

Annalisa Calini (College of Charleston)

Integrable flows for Legendrian curves in the 3-sphere related to the modified Korteweg-De Vries (mKdV) equation
I will describe a family of integrable geometric flows for closed Legendrian curves in the three-dimensional sphere, endowed with the standard contact structure defined by the distribution of tangent planes orthogonal to the circular fibers of the Hopf fibration. This structure is preserved by an action of the group SU(2) leading to invariant moving frame and curvature function for every regular Legendrian curve.

The curve flows are Hamiltonian with respect to a simple symplectic structure on the space of closed Legendrian curves, with the Hamiltonian vector field for the total length inducing the mKdV equation for the curvature. In general, the full mKdV hierarchy can be induced by geometric flows for Legendrian curves. If time allows, I will describe some special solutions, such as stationary solutions (evolving by rigid motions in the 3-sphere) and those generated by Legendrian lifts of circles in the 2-sphere.

This is joint work with Tom Ivey (College of Charleston) and Emilio Musso (Politecnico di Torino)

7 Abstracts Friday 11th

Marco Zambon (KU Leuven)

Coisotropic branes in symplectic geometry
A brane in a symplectic manifold M is a coisotropic submanifold N together with a closed 2-form which is compatible in a specific sense. This notion arises naturally from generalized complex geometry. We will first consider the case N=M (space-filling branes), i.e. the case in which M carries a holomorphic symplectic form. We will present some results on the deformations of the brane structure, i.e. deformations to nearby holomorphic symplectic forms having the same imaginary part.

For branes supported on lower-dimensional submanifolds, we then address the question of whether all coisotropic submanifolds nearby a given brane are themselves brane. We will elaborate on why the answer is negative in general, and provide an example.

This talk is based on ongoing work with Charlotte Kirchhoff-Lukat.

László Fehér (University of Szeged and Wigner Research Centre for Physics, Budapest)

Bi-Hamiltonian structures of spin Sutherland models

We review our results on bi-Hamiltonian structures of spin Sutherland models built on collective spin variables. Our basic observation was that the holomorphic cotangent bundle $T^*\mathrm{GL}(n,\mathbb{C})$ and its real form $T^*\mathrm{U}(n)$, as well as $T^*\mathrm{GL}(n,\mathbb{C})_{\mathbb{R}}$, carry a natural quadratic Poisson bracket, which is compatible with the canonical one. The quadratic bracket arises by change of variables and analytic continuation from an associated Heisenberg double. Then the reductions of $T^*\mathrm{GL}(n,\mathbb{C})$ and $T^*\mathrm{U}(n)$ by the conjugation actions of the corresponding groups lead to the holomorphic and real trigonometric spin Sutherland models, respectively, equipped with a bi-Hamiltonian structure. The reduction of $T^*\mathrm{GL}(n,\mathbb{C})_{\mathbb{R}}$ by the group $\mathrm{U}(n) \times \mathrm{U}(n)$ gives a generalized Sutherland model coupled to two $\mathfrak{u}(n)^*$-valued spins. We also show that a bi-Hamiltonian structure on the associative algebra $\mathfrak{gl}(n,\mathbb{R})$ that appeared in the context of Toda models can be interpreted as the quotient of compatible Poisson brackets on $T^*\mathrm{GL}(n,\mathbb{R})$. All these reductions were studied previously using the canonical Poisson structures of the cotangent bundles, without realizing the bi-Hamiltonian aspect.

Leendert Los (University of Groningen)

On the genericity of topology changes in level sets of proper Morse functions when passing critical points of index half the dimension of the manifold
Classical Morse theory describes how the topology of sublevel sets of a Morse function changes on variation of the level. Here we will consider topology changes of the level sets themselves.

Let us consider a smooth n-dimensional manifold M without boundary, with a proper Morse function f on it. If there are no critical points between two regular levels of f, then those levels are diffeomorphic, so we want to know what could happen when the level passes a critical value. Recently, A. Knauf and N. Martynchuk showed that when passing a critical point of index k unequal to $n/2$, and no other critical points of index $k-1$, $k+1$, or $n-k$, there is a change in the homotopy type of the level set. [1]

In this talk, we consider what could happen on a $2m$-dimensional manifold in the middle dimension: what can we say about topology changes in the level sets when passing a critical point of index m?

It is easy to find examples for $2m = 2$ where the topology does not change. One might guess that this is caused by nonorientability of M, but a similar example on $\mathbb{C}P^2$ shows that the situation is more complicated.

We will give conditions under which a change in homotopy type of level sets is guaranteed. One of our results states that if $H_m(M, Z_2)$ is zero, then the phenomenon of topology change of level sets at critical points is generic, in the sense that if f is a proper Morse function on M such that its critical points are on different levels, then the homotopy type of the regular level sets of f always changes on passing through one of those critical levels.

[1] Knauf, A., and Martynchuk, N., "Topology change of level sets in Morse theory", Ark. Mat., 58 (2020), 333-356.

OWEN DEARRICOTT (LA TROBE UNIVERSITY)

Integrable systems, Painlevé VI and explicit solutions to the anti-self dual Einstein equation via radicals.

Though Einstein's equation is well studied, relatively few Einstein metrics have been written in terms of explicit formulae via radicals. In this talk we discuss many such examples that occur as anti-self dual Einstein metrics and describe their singularities.

The construction heavily relies upon the theory of isomonodromic deformation and related algebraic geometry developed by N.J. Hitchin in the 1990s and the equivalence of the anti-self dual Einstein equation to a certain Painlevé VI equation under some symmetry assumptions discovered by K.P. Tod. The solution to Painlevé VI is achieved through a relation of its solution to pairs of conics obeying the Poncelet's porism by exploiting Cayley's criterion.

In this talk we discuss some important cases that are not well fleshed out in the literature, such as the solution of Painlevé VI associated with the Poncelet porism where the inscribing-circumscribing polygons have an even number of sides.

Moreover, we provide some explicit metrics with neutral signature and others with unusual cone angle singularities along a singular real projective plane that were speculated about by Atiyah and LeBrun.

ANUP ANAND SINGH (UNIVERSITY OF LEEDS)

Lagrangian multiforms on coadjoint orbits

Lagrangian multiforms provide a variational framework for describing integrable hierarchies using a generalised variational principle applied to an appropriate generalisation of a classical action. Much progress has been achieved since they were first introduced in a discrete setup in 2009. In this talk, I will report some recent results, based on a joint work with V. Caudrelier and M. Dell'Atti, for the case of Lagrangian 1-forms that covers finite-dimensional integrable systems.

We use the theory of Lie dialgebras, due to Semenov-Tian-Shansky, to construct a general Lagrangian 1-form living on a coadjoint orbit. Lie dialgebras are related to Lie bialgebras but are more flexible in that they incorporate the case of non-skew-symmetric r-matrices. The more famous Adler-Kostant-Symes scheme is, in fact, a special case of the Lie dialgebra construction.

We show that the Euler-Lagrange equations for our multiform produce the set of compatible equations in Lax form associated with the underlying r-matrix of the Lie dialgebra. We also give a structural result relating the closure relation for our multiform to the Poisson involutivity of Hamiltonians and the "double zero" on the Euler-Lagrange equations. Using the examples of the open Toda chain and the rational Gaudin model, we illustrate these results and the wide scope of our construction.

Andrey Mironov (Sobolev Institute of Mathematics)

Orthogonal curvilinear coordinate systems and torsion free sheaves on singular spectral curves

Classification and construction of orthogonal curvilinear coordinate systems in $\mathbb{R}^n$ is a classical problem of differential geometry. Using the Baker-Akhiezer function on smooth spectral curves, Krichever found a class of orthogonal curvilinear coordinate systems in $\mathbb{R}^n$. In this construction, the coordinate functions are expressed in terms of theta functions of the Jacobi varieties of spectral curves. We extend Krichever's construction to the case of reducible rational spectral curves equipped with torsion-free sheaves. In this case, the coordinate functions of orthogonal curvilinear coordinate systems are expressed in terms of elementary functions. Results were obtained with A.Senninger and I.A.Taimanov.

Yuri Suris (TU Berlin)

Geometric constructions of integrable birational maps

I will present several beautiful constructions from projective and algebraic geometry, leading to integrable birational maps of $\mathbb{P}^2$ and $\mathbb{P}^3$. Some of these maps are integrable discretizations of integrable continuous time systems, which remained elusive for years, till the geometry led to decisive breakthroughs.

8 Where to have lunch

These restaurants are located less than 15 minutes walking distance from the lecture hall. There are also many restaurants around Groenplaats and the cathedral of Antwerp (see map in Section 1.6).

ASIAN RESTAURANTS

Tasty Thai Express, *Kipdorpbrug 2*

Zaowang, *Oude Koornmarkt 22*
Sushi restaurant near the cathedral.

Aloha poké bowl and Woke bowl, *Jezusstraat 32*

Little BÚN - Vietnamese Noodle Shop - MAS, *Sint-Aldegondiskaai 44*
(Closed on Mondays)

BURGERS

Hard Rock Cafe, *Groenplaats 35*
They serve burgers and American food.

Manhattan's Burgers, *Groenplaats 1*
They serve burgers.

BRASSERIES

Brasserie De Post, *Groenplaats 26*

Horta Grand Café & Art Nouveau Zaal, *Hopland 2*

ITALIAN RESTAURANT

Da Giovanni, *Jan Blomstraat 8*
Close to the cathedral and there is plenty of space. There is a discount on the food if you have a student card.

SANDWICHES, SALADS, SOUPS

The Foodmaker, *Meir 27*
They serve salads and sandwiches. There is plenty of space.

Lunch Garden Antwerpen Inno Meir, *Meir 82*
This restaurant is located on the 4th floor of the shop Inno Meir. This is like a cafeteria.

Kool & Zo, *Prinsstraat 40*
This is a salad bar.

Panos Antwerpen Meir, *Meir 24*
This is a sandwich bar. They also serve salad and soup.

Foodvibes, *Minderbroedersrui 52*
They serve pastas and salads. On Monday and Tuesday it opens only at 6pm.

Le Pain Quotidien, *Graanmarkt 6*
Bakery where they also serve soups and tartines for lunch. There is enough space to sit.

Wasbar, *Melkmarkt 17*
They serve bagels and brunches and there is enough space to sit. You can also do your laundry there...

9 Activities (touristic and otherwise)

Here is a list of possible tourist activities that you could visit during your time in Antwerp. Please note that some of the museums require pre-booking a reservation.

Museum Plantin-Moretus, *Vrijdagmarkt 22*
Historic museum dedicated to the history of the printing, home of one of the oldest printing presses in the world.
Open until: 17:00, Price: €12.

Antwerpse Brouw Compagnie, Brouwerij & Taproom, *Indiestraat 21*
Brewery in the north part of the city that makes several popular local beers and serves food (both snacks and meals). Beer suggestion: Seef.
Open until: 23:00 (closed on Mondays and Tuesdays).

De Koninck - Antwerp City Brewery, *Mechelsesteenweg 291*
Local brewery founded in 1833. There is also a restaurant attached, and they offer tours of the brewery. Beer suggestion: Bolleke.
Open until: 19:00 (open until 10pm on Fridays and Saturdays).

The Chocolate Line, *Meir 50*
Chocolate shop which makes traditional chocolates as well as more contemporary experiments (including chili powder, Pop Rocks, etc...)
Open until: 18:30.

MAS, *Hanzestedenplaats 1*
Museum dedicated to the history, art and culture of Antwerp as a harbor city. This building, which is itself a piece of art, provides a free panoramic view over the city (even without visiting the museum).
Open until: 17:00, Price: €10 (but view from the top is free), online reservation advised.

Onze-Lieve-Vrouwekathedraal, *Groenplaats 21*
This cathedral is considered to be the heart of Antwerp.
Open until: 17:00, Price: €12.

Maagdenhuis, *Lange Gasthuisstraat 33*
Museum dedicated to the life of orphans and foundlings in Antwerp, located in the old orphanage for girls.
Open until: 17:00, Price: €8.

Red star line museum, *Montevideostraat 3*
Museum dedicated to people from Europe who tried to find a beter life in the USA and Canada in the 19th century.
Open until: 17:00, Price: €10, online reservation required.

ZOO Antwerpen, *Koningin Astridplein 20-26*
One of the oldest and most famous zoos of Europe.
Open Until: 18:00, Price: €32.50.

Sint-Andrieskerk, *Sint-Andriesstraat 5*
Late gothic church, famous for its naturalistic pulpit.
Open until: 17:00, Price: Free.

Sint-Jacobskerk, *Sint-Jacobstraat 9*
Church with the grave of the painter Rubens (the grave itself is currently closed).
Open until: 17:00, Price: free.

Middelheimmuseum, *Middelheimlaan 61*
Open-air museum in the Middelheimpark, containing statues, outdoor art, and a cafe.
Open until: 20:00, Price: Free.

Centraal Station Antwerpen, *Koningin Astridplein 27*
The Central Station of Antwerp, which is very close to the lecture hall, is a station with some extraodrinary beautiful architecture.
Price: Free.

ModeMuseum Antwerpen, *Nationalestraat 28*
Museum dedicated to fashion.
Open until: 18:00, Price: €12.

Koninklijk Museum voor Schone Kunsten Antwerpen, *Leopold de Waelplaats 1*
Museum with a collection of paintings, sculptures and drawings from the 14th–20th centuries.
Open until: 17:00, Price: €20, online reservation required.

Flandria, *Asiadok-Westkaai*
Boat trip in the Port of Antwerp. Website: `https://flandria.nu/en/homepage-en/`
Price: around €20 , reservation required (seats may sell out!).

10 List of participants

1. Dinmukhammed Akpan (Lomonosov Moscow State University)
2. Evgenii Antonov (Friedrich-Schiller University Jena)
3. Arnab Goswami Arnab Goswami (UM-DAE Centre for Excellence in Basic Sciences)
4. Maxim Arnold (The University of Texas at Dallas)
5. Salma Arrahmani (Faculty of Sciences Ibn Tofail Kenitra Morocco)
6. Victor Bangert (Universität Freiburg)
7. Silvan Bernklau (University of Jena)
8. Michael Bialy (Tel Aviv University)
9. Alexey Bolsinov (Loughborough University)
10. Gil Bor (CIMAT, Guanajuato, Mexico)
11. Dennis Borisov (University of Windsor, Canada)
12. Soufiane Boumasmoud (Cadi Ayyad University)
13. Bram Brongers (University of Groningen)
14. Joaquim Brugués (UAntwerpen/UPC)
15. Edward Bryden (Universiteit Antwerpen)
16. Annalisa Calini (College of Charleston)
17. Dahye Cho (Yonsei University)
18. Ioana Ciuclea (West University of Timisoara)
19. Roberto de Santana (University of Münster)
20. Owen Dearricott (La Trobe University)
21. Vladimir Dragovic (The University of Texas at Dallas)
22. Andrew Edwards (Case Western Reserve University)
23. Konstantinos Efstathiou (Duke Kunshan University)
24. Laszlo Feher (University of Szeged and Wigner Research Centre for Physics, Budapest)
25. Naomi Flamand (Universiteit Antwerpen)
26. Allan Fordy (Leeds)
27. Yuyang Gao (Loughborough)
28. Johan Gielis (University of Antwerp)
29. Tobias Våge Henriksen (University of Groningen)
30. Sonja Hohloch (University of Antwerp)
31. Andrew Hone (University of Kent)
32. Constant Huiszoon (University of Antwerp)
33. Senne Ignoul (Universiteit Antwerpen)
34. Anton Izosimov (University of Arizona)
35. Connor Jackman (University of Heidelberg)
36. Božidar Jovanović (Mathematical Institute SANU)
37. Melike Kaplan (Kastamonu University)
38. Giorgi Khimshiashvili (Ilia State University)
39. Jinhong Kim (Chosun University)
40. Joonhyung Kim (Chungnam National University)
41. Wookyung Kim (Lancaster University)
42. Andrei Konyaev (Moscow State University)

43. Boris Kruglikov (University of Tromso)
44. Leendert Los (University of Groningen)
45. Krzysztof Marciniak (Linköping University)
46. Ian Marquette (The University of Queensland)
47. Nikolay Martynchuk (University of Groningen)
48. Vladimir S. Matveev (Friedrich Schiller University of Jena)
49. S. Habib Mazharimousavi (Eastern Mediterranean University)
50. Tom Mestdag (University of Antwerp)
51. Andrey Mironov (Sobolev Institute of Mathematics)
52. Richard Montgomery (UC Santa Cruz)
53. Zouhair Mouayn (Sultan Moulay Slimane University)
54. Leonard Mushunje (Columbia University)
55. Joseph Palmer (University of Antwerp & UIUC)
56. Marco Pollanen (Trent University)
57. Giuseppe Pucacco (University of Rome Tor Vergata)
58. Manuel Quaschner (Friedrich Schiller University Jena)
59. Giovanni Rastelli (University of Turin)
60. İzzet Sakalli (Eastern Mediterranean University)
61. Pedro Santos (University of Antwerp)
62. Anup Anand Singh (University of Leeds)
63. Wijnand Steneker (Universitetet i Tromsø)
64. Stiven Ramirez Quintero (IME USP)
65. Yuri Suris (TU Berlin)
66. Sergei Tabachnikov (Pennsylvania State University)
67. Susan Tolman (UI Urbana-Champaign)
68. Daniel Tsodikovich (Tel Aviv University)
69. Pierre Van Moerbeke (UC Louvain)
70. Mats Vermeeren (Loughborough University)
71. Cornelia Vizman (West University of Timisoara)
72. Andreas Vollmer (University of Hamburg)
73. Aldo Witte (University of Antwerp)
74. Kenzo Yasaka (University of Antwerp)
75. Marco Zambon (KU Leuven)