Geometric Gauges:

Plane and Simple

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Euclidean geometry, while familiar, reveals profound insights when viewed through the lens of Geometric (i.e. Clifford) Algebra. This talk explores how Geometric Algebra naturally captures the essence of flat geometry, if we invert our traditional perspective. Instead of vectors representing arrows pointing at points, grade-1 vectors correspond to (d-1)-dimensional hyperplanes: the very subspaces left invariant by reflections. This "upside-down" view elegantly aligns with the Cartan-Dieudonné theorem, demonstrating that products of vectors encompass all possible isometries.

Furthermore, this plane-based approach directly links the invariants of isometries to geometric elements like points, lines, planes, etc., echoing the core principles of Klein's Erlangen Program. Conversely, these geometric entities act as isometries themselves, namely as point-reflections, line-reflections, plane-reflections, etc. This exposes a beautiful relationship between isometries and the elements upon which they act: they are one and the same.

A key result, the invariant decomposition theorem, states that any composition of k linearly independent reflections can be decomposed into floor(k/2) commuting factors, each being the product of at most two reflections. Since the product of two reflections results in a rotation, translation, or boost, this implies there are no fundamentally new isometries after the bireflections (products of two reflections).

However, determining the canonical bireflections for a given isometry had long remained a challenge. This talk presents both geometric and algebraic solutions to this problem, with geometric gauges taking center stage.

Lastly, we will proceed to discuss how these principles change our notion of what a spinor representation is and will discuss several applications in mathematical physics.