# Billiard tables with rotational symmetry

Daniel Tsodikovich
Tel Aviv University

Joint work with Misha Bialy

Finite Dimensional Integrable Systems 2023
Antwerp

August 2023

# Motivation

The goal of this talk is to generalize, in various ways, the following simple claim:

**Claim**

N®O}´ HOj§}9ee´ ´h hO§}_H qe9j9} Hnj®O² @nL´ nT Hnj §9j§
°_L§\ _ 9 L_ cu

And we will see the relation to the following fact:

**Theorem (Cyr)**

Sn} 2 j 2 NJ §\0 0x«9§_nj tan(j ²) = j tan(²) \9 jn
ne«§_nj Tn} ² 2 Q n Zu

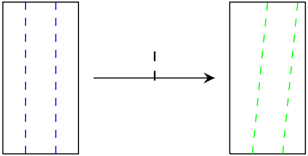
# Outline

- Introduction - billiards and twist maps
- Formulation of the main results
- Ideas of proofs

# Introduction to twist maps of cylinders

$A = \mathbb{S}^1 \times \mathbb{R}$ - cylinder. $\varphi : A \to A$ - area preserving diffeomorphism.

- $\varphi(x, q) = (w(x, q), p(x, q))$ is called a **twist map**, if $\frac{\partial w}{\partial q} \neq 0$ (**twist condition**).
- $(\tilde{w}(\tilde{x}, q), p(\tilde{x}, q)) = \tilde{\varphi}(\tilde{x}, q)$ - lift of $\varphi$ to $\mathbb{R}^2$.
- Alternative formulation of the twist condition: for every fixed $\tilde{x}_0 \in \mathbb{R}$, the function $q \mapsto \tilde{w}(\tilde{x}_0, q)$ is monotone.



# :2M2` iBM; 7mM+iBQMb Q7 irB

- h?2 7 +i i h?Bib `2 T`2b2`pBM; K2 Mb i?
  /S^ /Z= /T^ /[() / S/Z T/[= yX

# :2M2` iBM; 7mM+iBQMb Q7 irB

- h?2 7 +i i?Bib `2 T`2b2`pBM; K2 Mb i? /S^ /Z= /T^ /[() / S/Z T/[= yX
- q2 b v i?hiBb 2M +i irBKi T B7 i?2`2 2tBbi 7mM+iBQM R U2M2` iBM; 7mMQiBQM +? S/Z T/[= /a- M(~[+ RZ+ R= a(~[;Z) X

# :2M2` iBM; 7mM+iBQMb Q7 irB

- h?2 7 +i i h?Bib `2 T`2b2`pBM; K2 Mb i?
  /S^ /Z= /T^ /[() / S/Z T/[= yX
- q2 b v i?hiBb 2Mt +i irBKi T B7 i?2`2 2tBbi
  7m M+iBQM R U; 2M2` iBM; 7mM+iBQM Q7 irBbi
  S/Z T/[= /a- M a(~[+ R Z+ R= a(~[; Z) X
- h r B b i +Q M / B i B Q M B b @ @[a ?B p BHM2;M i+iQ M b i
  b B ; M@X/ 2UM Q i 2 b T `i B H /2` B pp i BB 2# HX2`VX i

# :2M2` iBM; 7mM+iBQMb Q7 irB

- h?2 7 +i i?Bib `2 T`2b2`pBM; K2 Mb i? /S^ /Z= /T^ /[() / S/Z T/[= yX
- q2 b v i?hiBb 2M +i irBKi T B7 i?2`2 2tBbi 7mM+iBQM R !M R U; 2M2` iBM; 7mM+iBQM Q7 i?B +? S/Z T/[= /a- Ma(~[+ R Z+ R= a(~[; Z) X
- hrBbi +QM/BiBQM Bb @[a?B p BH2;Mi+iQMbi bB; M@X/ 2UM Qi2b T `iB H /2` B pp iBBp 2# HX2`VXi
- irBbi K T + M #2 /2}M2/ mbBM; ;2M2` #v b2iiBM;

$$\hbar(\sim[; T) = ( Z; S) () \qquad ( \; T = @_R a(\sim[; Z) \;;\; S = @_k a(\sim[; Z)$$

# o `B iBQM H b2iiBM;

- b2[m2f $M_{[N}$ g2 $_{M}$ $_{Z}$ R Bb + HH2/ Q/M};m` iQ1 ?QBR7 i?2`2 2ft Bbib $_{M}$ $_{Z}$ R bm+? i(?; iT) = ~h(~[; T) :

# o `B iBQM H b2iiBM;

- b2[m2fM{g2 $_{[M]\,M\,Z}$ R Bb + H HQ/M};m` iBQM B7 i?2`2 2ft Bgbi $_{M\,M\,Z}$ R bm+? i?$(\tilde{[}_M; T_M) = \hat{h}^M(\tilde{[}_y; T_y)$:
- b2[m2fM{g2 $_{[M]\,M\,Z}$ R Bb +QM};m` iBQM B7 7Q` JH<HL2 Z - i?2 TQBMi $(\tilde{[}_{J+R}; \ldots; \tilde{[}_{L\,R})$ Bb +`BiB+ T QBMi Q7 iiB?QM 7mM+iBQM H

$$6_{J;L}(t_{J+R}; \ldots; t_{L\,R}) =$$

$$a(\tilde{[}_J; t_{J+R}) + \sum_{M=J+R}^{k} a(t_M; t_{M+R}) + a(t_{L\,R}; \tilde{[}_L):$$

# o `B iBQM H b2iiBM;

- b 2[m 2fM[ g 2 R Bb + H HQ/M};m` iBQM B7 i?2 `2 2ft Bgbi R bm+? i(?[M; iTM) = ~h M(~[y; Ty):
- b 2[m 2fM[ g 2 R Bb +QM};m` iBQM B7 7Q` JHL2 Z - i?2 TQBMi(~[J+R; :::; ~[L R) Bb +`BiB+ TQBMi Q7 i?2 7mM+iBQM H

$$6_{J;L}(t_{J+R}; :::; t_{L R}) = a(~[_J; t_{J+R}) + \sum_{M=J+R}^{k} a(t_M; t_{M+R}) + a(t_{L R}; ~[_L):$$

- +QM};m` iBQMKBbM+B HKBH2 7M2p2`v }MBi2 b2;K2Mi Bb HQ+ H KBMBKmK Q7 i?2 +

# AKTQ`i Mi i?2Q`2Kb #Qmi irB

`Qi iBQM H BMp `B Mi +m`p2 Q7+mi`p2bi KBTb +m`p2 A2 r?B+? Bb MQi +QMi` +iB# H2 (h) = M /X7Q` r?B+?

**h?2Q`2K U"B`F?Qzöb h?2Q`2KV**

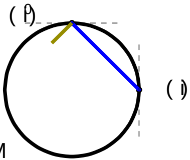
1p2`v `Qi iBQM H BMp `B Mi +m`p2 Q7 M `2 T`2b ;` T? Q7 GBTb+?Bix7 mMX+iBQM 7

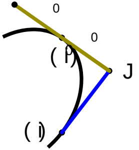
**h?2Q`2K**

G2i :hA!A #2 M 2t +i irBbi K T-#2M/MH2BiMp `B Mi +m h X h?2M Mv [Q`#Bi? i Bb bbQ+B i2/ iQ MBbTQBMi KBMBKBxBM;X

# "BHHB `/b birBbiK Tb

| "BHHB `/ ivT2? b2 | +vHBM/2` M/ ;2M2` iBM; 7mMB+iBQM |
|---|---|
| "B`F?Qz | f( i; cos ) j 2 ( y; )g<br>i @ `+ H2M;i? T ` K2i2` QM<br>a( i; ') = j ( i) ( ')j |
| Pmi2` | f( i; ) j > yg<br>J = ( i) + _( i) = ( ') 0_( ')<br>a( i; ') = `2( + Q(M[pf Jg))<br>S `iB H /2`BpaiB`p2 2`b2 Q 7i2/ MQ/ 0X |



# "BHHB `/b b irBbi K Tb @ +QM

"BHHB `/ ivT2S? b2 +vHBM/2` M/ ;2M2` iBQM; 7 bmH+iimB`Q2M

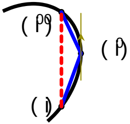
avKTH2+iB+ h( i; b) = ( ρ; θ) ()

f( i; b)g

b = det(_( i); ( ρ));

θ = det( ( i); _( ρ))

a( i; ρ) = det( ( i); ( ρ))



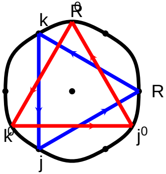
JBMFQrbFB rX`Xi MLQ`K h( i; b) = ( ρ; θ) ()

f( i; b)g

b = /L ( ρ) ( i)( _( i))

θ = /L ( ρ) ( i)(_( ρ))

a( i; ρ) = L( ( ρ) ( i))



- "B`F?Qzöb +QMD`2+im`2 Q7 #BHHB `/ b v QMHv BMi2;` #H2 UK2 MBM;- i?2 T? b2 `Qi iBQM H BMp `B Mi +m`p2bV #BHHB

- "B`F?Qzöb +Q7MQD`2+Bi`m?2Qz #BHHB `/ b v QMHv BMi2;` #H2 UK2 MBM;- i?2 T? b2 `Qi iBQM H BMp `B Mi +m`p2bV #BHHB
- Ai r b b?QrM i? i BM "B`F?Qz- Pmi2`- M #BHHBiQ`i/ b- BMi2;`Ü B#X2HX- i7 QHB iBQM Q7 +vHBM/2` #v BMp `B Mi +m`p2bV Bb `B; " ` ++Q "2`M `/BVX

- "B`F?Qzöb +QMD2+im`2 7Q` "B`F?Qz #BHHB `/ b v QMHv BMi2;` #H2 UK2 MBM;- i?2 T? b2 `Qi iBQM H BMp `B Mi +m`p2bV #BHHB
- Ai r b b?QrM i? i BM "B`F?Qz- Pmi2`- M #BHHB `/b Q` i/ H- BMi2;` #BHHB `/b i?2 7QHB iBQM Q7 +vHBM/2` #v BMp `B Mi +m`p2bV Bb `B; " ` ++Q "2`M `/BVX
- h?2 rQ`F Q7 "B Hv M/ JB`QMQp b?Qrb i bvKK2i`v bbmKTiBQM QM i?2 i #H2- Bi bbmK2 i? i QMHv bT2+B H T `i Q7 i?2 7QHB i2/ #v BMp `B Mi +m`p2bX

- "B`F?Qzöb +QMD2+im`2 7Q` "B`F?Qz #BHHB `/ b v QMHv BMi2;` #H2 UK2 MBM;- i?2 T? b2 `Qi iBQM H BMp `B Mi +m`p2bV #BHHB
- Ai r b b?QrM i? i BM "B`F?Qz- Pmi2`- M #BHHB`/b- BMi2;` #H2 #BHHB`/b iQ i?2 7QHB iBQM Q7 +vHBM/2` #v BMp `B Mi +m`p2bV Bb `B; " ` ++Q "2`M `/BVX
- h?2 rQ`F Q7 "B Hv M/ JB`QMQp b?Qrb i bvKK2i`v bbmKTiBQM QM i?2 i #H2- Bi bbmK2 i? i QMHv bT2+B H T `i Q7 i?2 7QHB i2/ #v BMp `B Mi +m`p2bX
- q2 rBHH +QMbB/2` ?B;?2` Q`/2` bvKK2i` b2iiBM; 7Q` Qi?2` #BHHB `/ bvbi2KbX

- "B`F?Qzöb +QMD2+im`2Qz #BHHB `/ b v QMHv BMi2;` #H2 UK2 MBM;- i?2 T? b2 `Qi iBQM H BMp `B Mi +m`p2bV #BHHB
- Ai r b b?QrM i? i BM "B`F?Qz- Pmi2`- M #BHHB `/ b BMi2;` #H2 i? i BM "B`F?Qz- Pmi2`- M #BHHB `/ 7QHB iBQM Q7 +vHBM/2` #v BMp `B Mi +m`p2bV Bb `B; " ` ++Q "2`M `/BVX
- h?2 rQ`F Q7 "B Hv M/ JB`QMQp b?Qrb i bvKK2i`v bbmKTiBQM QM i?2 i #H2- Bi bbmK2 i? i QMHv bT2+B H T `i Q7 i?2 7QHB i2/ #v BMp `B Mi +m`p2bX
- q2 rBHH +QMbB/2` ?B;?2` Q`/2` bvKK2i` b2iiBM; 7Q` Qi?2` #BHHB `/ bvbi2KbX
- h?2b2 `2bmHib /Q MQi bbmK2 7QHB iB #mi ` i?2 2tBbi2M+2 Q7 QM2 bT2+B

# 6Q`KmH iBQM Q7 i?2 `2bmHib

G2i #2 bKQQi? TH M ` bi`B+iHv +QMp2t +m`p2X

# 6Q`KmH iBQM Q7 i?2 `2bmHib

G2i #2 bKQQi? TH M ` bi`B+iHv +QMp2t +m`p2X

**h?2Q`2K R**

A7 Bb BMp `B Mi mM/2` i?2 `Qi iBQM Q7 $\frac{k}{F}$ i?2M ... /2` i?2 "B`F?Qz #BHH
K T BM? b M BMp `B Mi +m`p2 Q7 F T2`BQ/B+ Q`#Bi X i?2M

# 6Q`KmH iBQM Q7 i?2 `2bmHib

G2i #2 bKQQi? TH M ` bi`B+iHv +QMp2t +m`p2X

## h?2Q`2K R

A7 Bb BMp `B Mi mM/2` i?2 `Qi iBQM $\frac{k}{F}$ B QBMi?# F j-M ;MH/2i?2 "B`F?Qz #BHH
K T BM? b M BMp `B Mi +m`p2 Q7 F T2`BQ/B+ +Q``#BHH2X i?2M

## h?2Q`2K k

A7 Bb BMp `B Mi mM/2` Mj 2QH`/2K`2F:iG(k7R) - M/ 2Bi?2` i?2 Pmi2` Q
avKTH2+iB+ #BHHB?`/pk2 TMQB7Mp `B Mi +m`p2 Q7 F T2`BQ/B+ MQ`#
2HHBTb2X

# 6Q`KmH iBQM Q7 i?2 `2bmHib

G2i #2 bKQQi? TH M ` bi`B+iHv +QMp2t +m`p2X

**h?2Q`2K R**

A7 Bb BMp `B Mi mM/2` i?2 `Qi i BQM Q7 $\frac{k}{F}$ i?2M QMi?2 #Fj-M ;MH/2i?2 "B`F?Qz #BHH
K T BM? b M BMp `B Mi +m`p2 Q7 F T2`BQ/B+ +Q `#Bi2X i?2M

**h?2Q`2K k**

A7 Bb BMp `B Mi mM/2` j M 2QH`2K`2MiG(k,R) - M/ 2Bi?2` i?2 Pmi2` Q
avKTH2+iB+ #BHHB?`/pk TMQB7Mp `B Mi +m`p2 Q7 F T2`BQ/B+ MQ`
2HHBTb2X

**h?2Q`2K j**

G2i #2 +m`p2 BMp `B Mi mM/2`j 2MHQ`K2MiG(k,R) X h?2 JBMFQrbF
#BHHB `/ K -T BQ7 i?2 MQ`K BM/m+2/ #v M BMp `B Mi +m`p2 Q7 F
Q`#Bib- B7 M/ QMHv B7 `B Mi mM/2` M: 2(k,R) 2MiQ`72` F r?2`2

$$
\begin{cases}
= R; \text{ B7 } Fk \pmod 9; \\
= k; \text{ B7 } Fy \pmod 9; \\
= 9; \text{ B7 } FR \pmod k:
\end{cases}
$$

# *Q`QHH `B2b

- :Bp2M UbvKK2i`B+V QMk−K M/ irQ mMBi p2+iQ`b i?viBb "B`F?Qz Q`i?QQM H iQ 2 ?y Bb i?2 HQ+ H KBMBKmK Q7 i?2 7mM+ i7!k t+ ikX
- h?2 MQ`K Bb + HH2/ Q`K B7 i?2 "B`F?Qz Q`i?Q;QM HBiv `2H bvKK2i`B+X
- Ai + M #2 b?QrM i? i i?2 mMBi +B`+H2 Q7 _ /QM MQ`K ? b 9 T2`BQ/B+ #BHHB `/ Q`#Bib 7Q` i?2 Pmi2` #BHHB `/ K TX

***Q`QHH `v**

_ /QM MQ`K R k BM?B+? Bb BMp `B Mi mM/2` HBM2 ` K T Q7 Q`/2 1m+HB/2 M MQ`KX

_2K `F #Hv- MQM@1m+HB/2 M M HviB+ _ /QM MQ`Kb F?7pQBM; b F jQ// ? p2 #22M +QMbi`m+i2/ #v "B Hv- "Q`- M/ h # +?MBFQp

# *Q`QHH `B2b

irBbi K T Bb b B/Q iiQH#H2v BMi2;` #H2 BMQ7M
i?2 T? b2 +vHBM/2` 7Q`K B iBQM #v BMp `B
i?2 irBbi K TX Ai2` iBM;;Bhp?22bQ`2K

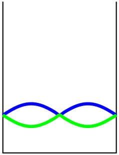
## *Q`QHH `v

G2i #2 k*@bKQQi?- TH M `- bi`B+iHv +QMp
BMp `B Mi mM/2` HBM2 `jKX T*QM7 bQB`//2` iF?2
JBMFQrbFB #BHHB-`r/Bbi?bi?22KMBQM`K BMX/mA+72
i?2 JBMFQrbFB #BHHB i`Q K HTHQv 7BMi2;` #H
2MiB`2 T? b2 +vHBMb/2`U-1im?2+MHB/2 MV 2HHB

# hrBbi K T `;mK2Mi mb2/ BM H

## S`QTQbBiBQM

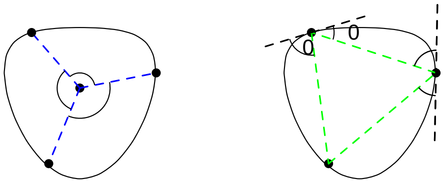
G2i h #2 M 2t +i irBbi K TX $_R$ b $_k$ m`K2 i ni Q i `Qi iBQM H BMp `B Mi +m`p2b Q7 h- Q7 i?2 MmK#2`- M/ i? i HH TQBMib Q7 i?Qb2 +m Q7 hX h?2 $_R$ 2 M $_k$ X



Ai Bb TQbbB#H2 iQ ? p2 b2p2` H BMp `B Mi +m`p2b Q7 i?2 b K2 M 2HHBTb2V- #mi MQi B7 HH TQBMib Q7 i?Qb2 +m`p2b `2 T2

# *QM+HmbBQM 7`QK i?Bb `;mK

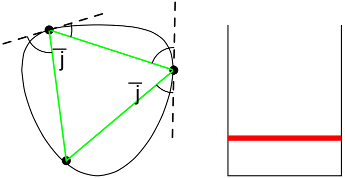
AM HH 7Q`m` bvbi2Kb- i?2 bvKK2i`v Q7 i?2
;Bp2M BMp `B Mi +m`p2 iQ MQi?2` 2B F2Q/ B
Q`#BibX "v i?2 S`QTQbBiBQM Bi Kmbi +QB
QM2X 1p2Mim HHv- i?Bb F2 `M2H Q/B +i #2 +HB
+QBM+B/2b rBi? M Q`#Bi Q7 i?2 bvKK2i`vX




7i2` i? i- 2 +? ivT2 Q7 #BHHB `/ bvbi2K Bb

# "B`F?Qz * b2 @ :miFBM *QM/B

AM i?2 + b2 Q7 "B`F?Qz #BHHB `/b- r2 +QM
Q`#Bi 7Q`Kb `2;mH` TQHv;QM- M/ ?2M+
Q7 i?2 Q`#Bi `2 +QMbi MiX b `2bmHi i?2
:miFBM +QM/BiBQM rBi? M M;H2 r?B+? B
X




bbmKBM; Eb MQi +B`+H2- i?2 BM+B/2M+2
b iB tan(M) = M tan( ) 7Q` b QMKN2 - #mi ++Q`/BM;
i?2Q`2K #v *v`- i?B b Bb B2KTQb Bb2 #H2 7Q`

# Pmi2` "BHHB `/ `;mK2Mi

- bbmK2 i?Bib kT 2`BQ/B+X

# Pmi2` "BHHB `/ `;mK2Mi

- bbmK2 i?Bib kT2`BQ/B+X
- G2i #2 i?2 HBM2 ` K FiQ?7iQT`/2b`2Xp2b

# Pmi2` "BHHB `/ `;mK2Mi

- bbmK2 i?Bib kT2`BQ/B+X
- G2i #2 i?2 HBM2 ` K FiQ?7i QT`/22b`2 Xp2b
- h?2 T`2pBQmb `;mK2Mi BKTHB2b i? i i
  Q7 i?2 Pmi2` #BHHB `/ Q(`)# B`i2bi `iBM; i
  (i); (i); :::; F R(i) X

# Pmi2` "BHHB `/ `;mK2Mi

- bbmK2 i?Bib kT2`BQ/B+X
- G2i #2 i?2 HBM2 ` K FiQ 7i QT`/22b`2 Xp2b
- h?2 T`2pBQmb `;mK2Mi BKTHB2b i? i i Q7 i?2 Pmi2` #BHHB `/ Q(`)# B`i2bi `iBM; i ( i); ( i);:::; F R( i) X
- PM2 + M +?QQb2 i?2 QTM`BKM2bi2m`+? bQT?BbiB+ i2/ r v- bQ i? i i?2`2 ▷BH H # 7Q` r?B+?, UPmi2` #BHHB `/ H r Y S`QT

$$( i) + \_( i) = ( i) \_( i);$$

M/ BM //BiBQM-

$$( i) = ( i+ \frac{k}{F}):$$

# Pmi2` "BHHB `/ `;mK2Mi

- h?2 Pmi2` #BHHB `/ H r i?2M #2+QK2b,

$$(\_(i) + \_(i + \frac{k}{F})) = (i + \frac{k}{F}) \quad (i) :$$

# Pmi2` "BHHB `/ `;mK2Mi

- h?2 Pmi2` #BHHB `/ H r i?2M #2+QK2b,

$$(\_(i) + \_(i + \frac{k}{F})) = (i + \frac{k}{F}) \quad (i):$$

- q`Bi2 6Qm`B2` 2tT, $M(i) = \overset{P}{B} QM_{2Z} Q_{M2Z} 2^{BMi} X$

# Pmi2` "BHHB `/ `;mK2Mi

- h?2 Pmi2` #BHHB `/ H r i?2M #2+QK2b,

$$(\_(i) + \_(i + \frac{k}{F})) = (i + \frac{k}{F}) \quad (i):$$

- q`Bi2 6Qm`B2` 2tT, M(i) = B QM $_{M2Z}$ Q$_M$ 2$^{BMi}$ X
- *QKT `2 +Q2{+B2Mib BM i?2 #Qp2 7Q`K

$$+_M(2^{B\frac{kM}{F}} \quad R) = BM_M(2^{B\frac{kM}{F}} + R):$$

# Pmi2` "BHHB `/ `/ ` ;mK2Mi

- h?Bb K2 Mb i? i i2 ByiQ2à $\frac{M}{F}$ = MX

# Pmi2` "BHHB `/ `;mK2Mi

- h?Bb K2 Mb i? i 2 Byi Q2 $\tan \frac{M}{F} =$ M X
- ++Q`/BM; iQ aim`K@>m`rBix@E2HHQ;; i? i+R Q`+R `2 MQi x2`Q $\tan \frac{Q}{F}$ bQ b `2bmH Q#i BM i?  +M6 By7 i?2M r2 ? p@2` iBQM H bQH :miFBMöb 2[m iBQM,

$$\tan \frac{M}{F} = M \tan \frac{}{F}:$$

# Outer Billiard argument

- This means that either $H_j = 0$ or $\tan \frac{j}{c} = \quad j$.
- According to Sturm-Hurwitz-Kellogg theorem, it follows that $H_1$ or $H \;\; _1$ are not zero, so $\quad = \tan \frac{}{c}$, so as a result we obtain that if $H_j \neq 0$ then we have a -rational solution to Gutkin's equation:

$$\tan \frac{j}{c} = j \tan \frac{}{c} :$$

- As mentioned before, this is impossible.

# Outer Billiard argument

- This means that either $H_j = 0$ or $\tan \frac{j}{c} = \quad j$.
- According to Sturm-Hurwitz-Kellogg theorem, it follows that $H_1$ or $H_{\ 1}$ are not zero, so $\quad = \tan \frac{}{c}$, so as a result we obtain that if $H_j \neq 0$ then we have a -rational solution to Gutkin's equation:

$$\tan \frac{j}{c} = j \tan \frac{}{c} :$$

- As mentioned before, this is impossible.
- As a result, we must have $H_j = 0$ for all $jjj > 1$ which means that is an ellipse.

# Sketch of argument for Minkowski case

- d0§ $8$ @0 §\0 ´h h0§}´ nT §\0 §9@e0J n §\9§ §\0 g_jcn° c_ @_ee_9}L n}@_§ §\9§ §9}§ 9§ (§) _ (§); $8$ (§); :::; $8^c$ º (§)u
- ~_jH0 9ee n}@_§ §\9§ §9}§ T}nh §\0 9h0 _j®9}_9j§ H«}®0 9}0 h_j_h_¶0} J _§ Tneen° §\9§

$$\sum_{\_=o} X(8^- (§) \quad 8^- \, ^o (§))$$

_ Hnj §9j§ r$X$ _ §\0 jn}h °_§\ }0 q0H§ §n °_H\ _ §\0 «j_§ H_}He0su

- ~_jH0 _ _j®9}_9j§ «jL0} $8$ _§ Tneen° §\9§ $X(8$ (§) (§)) _ Hnj §9j§u
- ¦ \0}0Tn}0 $8$ (§) (§) _ 9e°9´ nj 9 »²0L \nhn§\0§_H Hnq´ nT u
- 8j9e´ _ nT §_ HnjL_§_nj e09L §n §\0 9LL_§_nj9e ´h h0§}´ nT u