Finite dimensional integrability in mathematical physics

SwissMAP Research Station at Les Diablerets
June 11-16, 2023

Workshop booklet

This workshop on *Finite dimensional integrability in mathematical physics* takes place at the SwissMAP Research Station at Les Diablerets in Switzerland during June 11-16, 2023. It is an follow-up event of the two workshops *Geometric aspects of momentum maps and integrability* (2018) and *Integrable systems* (2016) that took place at the CSF in Ascona/Switzerland.

Standard airports to fly to are Geneva/Switzerland (GVA) or Zurich/Switzerland (ZRH) from which you'll have to take the train/bus to get to the SwissMAP Research Station in Les Diablerets/ Switzerland. For detailed travel directions and information on the accommodation, please consult the webpage of the SwissMAP Research Station.

Organizers:

- Anton Alekseev (Geneva)
- Sonja Hohloch (Antwerp)
- Tudor Ratiu (Shanghai Jiao Tong University & EPFL)

Sponsors:

This event takes place at the SwissMAP Research Station and is thus sponsored by the University of Geneva, the ETH Zurich, the Swiss National Science Foundation, and SwissMAP. For details, please consult the sponsor information on the webpage of the SwissMAP Research Station.

Speakers:

- Jaume Alonso (TU Berlin): *abstract*
- Philip Boalch (Institut de Mathématiques de Jussieu-Paris Rive Gauche): *abstract*
- Alexey Bolsinov (Loughborough): *abstract*
- Holger Dullin (University of Sydney): *abstract*
- Konstantinos Efstathiou (Duke Kunshan University): *abstract*
- Maxime Fairon (Paris-Saclay): *abstract*
- Laszlo Feher (University of Szeged & Wigner Research Centre for Physics): *abstract*
- Anton Izosimov (University of Arizona): *abstract*
- Yohann Le Floch (Strasbourg): *abstract*
- Yanpeng Li (Sichuan University): *abstract*
- Nikolay Martynchuk (Groningen): *abstract*
- Eckhard Meinrenken (Toronto): *abstract*
- Yong-Geun Oh (IBS Center for Geometry and Physics & POSTECH): *abstract*
- Joseph Palmer (Urbana-Champaign & Antwerp): *abstract*
- Daisuke Tarama (Ritsumeikan University): *abstract*
- Susan Tolman (Urbana-Champaign): *abstract*
- Cornelia Vizman (West University of Timisoara): *abstract*
- San Vũ Ngọc (Rennes 1): *abstract*
- Aldo Witte (Antwerp): *abstract*

Participants:

- Evgenii Antonov (Jena)
- Nikolai But (Geneva)
- Alejandro Calleja Arroyo (Barcelona)
- Ioana Ciuclea (West University of Timisoara)
- Rea Dalipi (Geneva)
- Anfisa Gurenkova (Geneva)
- Tobias Vage Henriksen (Groningen)
- Constant Huiszoon (Antwerp)
- Leendert Los (Groningen)
- Damien McLeod (Sydney)
- Muze Ren (Geneva)
- Kenzo Yasaka (Antwerp)

Finite dimensional integrability in mathematical physics

Les Diablerets (June 11-16, 2023)

Sunday (June 11)	**Monday** (June 12)	**Tuesday** (June 13)	**Wednesday** (June 14)	**Thursday** (June 15)	**Friday** (June 16)
	Breakfast (served between 7:30 – 9:00)				
	9:00 – 9:50 **Bolsinov**	9:00 – 9:50 **Boalch**	9:00 – 9:50 **Vu Ngoc**	9:00 – 9:50 **Fairon**	9:00 – 9:50 **Tarama**
	10:00 – 10:30 **Coffee break**				
	10:30 – 11:20 **Alonso** 11:30 – 12:20 **Witte**	10:30 – 12:20 **Poster session**	10:30 – 11:20 **Efstathiou** 11:30 – 12:20 **Martynchuk**	10:30 – 11:20 **Li** 11:30 – 12:20 **Oh**	10:30 – 11:20 **Meinrenken**
	Lunch (starts at 12:30)				
	16:30 – 17:00 **Coffee break**	16:00 – 16:50 **Tolman**		16:30 – 17:00 **Coffee break**	
	17:00 – 17:50 **Feher** 18:00 – 18:50 **Vizman**	17:00 – 17:30 **Coffee break** 17:30 – 18:20 **Palmer**		17:00 – 17:50 **Izosimov** 18:00 – 18:50 **Dullin**	
Dinner 19:15 – 20:30	19:00 – 19:30 **Aperitif**	18:30 – 19:20 **Le Floch**			
	Dinner (starts at 19:30)				

Abstracts of Talks

(in alphabetical order)

Integrable birational discretisations from a 3D generalisation of QRT

Jaume Alonso

Institute of Mathematics, Technische Universität Berlin, Berlin, Germany
E-mail: alonso@math.tu-berlin.de

When completely integrable Hamiltonian systems are discretised, the resulting discrete time systems are often no longer integrable themselves. This is the so-called *problem of integrable discretisation*. Two known exceptions to this situation in 3D are the Kahan discretisations of the Euler top and the Zhukovski-Volterra gyrostat with one non-zero linear parameter β, both birational maps of degree 3. The integrals of these systems define pencils of quadrics. By analysing the geometry of these pencils, we develop a framework that generalises QRT maps and QRT roots to 3D, which allows us to create new integrable maps as a composition of two involutions. We show that under certain geometric conditions, the new maps become of degree 3. We use these results to create new families of discrete integrable maps and we solve the problem of integrability of the Zhukovski-Volterra gyrostat with two β's.

This is a joint work with Yuri Suris and Kangning Wei.

Modular integrable systems, their deformations and classification

Philip Boalch (CNRS, IMJ-PRG, Paris)

I'll discuss the moduli space approach to finite dimensional integrable systems admitting "good" rational Lax representations, as well as their deformations to isomonodromy systems. This viewpoint was initiated by Hitchin in the higher genus case with no poles and then extended by Nitsure/Bottacin/Markman to cover the cases with poles, and in particular to incorporate existing systems in the genus zero case (rational matrices) where many of the applications lie. A key step (Biquard-B. 2004) was to show such integrable systems have complete hyperkahler metrics on their total space, and so (by hyperkahler rotation), are canonically diffeomorphic to moduli spaces of meromorphic connections. This step amounts to a generalisation of non-abelian Hodge theory to incorporate poles of any order, and this rotation is a modern intrinsic way to understand the autonomous limit of an isomonodromy system to an isospectral integrable system (dating back to Painlevé and Garnier).

Finally one can also consider the moduli spaces of monodromy/Stokes data (the wild character varieties) corresponding to such spaces: we will view these as "global" analogues of Lie groups and explain some first steps in their classification, via diagrams, analogous to Dynkin diagrams. One of the simplest class of examples of wild character varieties (related to Painlevé II) are the Flaschka–Newell surfaces:

$$x\,y\,z + x + y + z = b - b^{-1}$$

where $b \in \mathbb{C}^*$ is constant. These complex surfaces underly the simplest examples of the wild nonabelian Hodge hyperkahler manifolds, and their diagram is affine A_1.

• O. Biquard and P.B., Wild non-abelian Hodge theory on curves, Compositio Math. 140 (2004), no. 1, 179–204.

• P.B., Symplectic manifolds and isomonodromic deformations, Adv. in Math. 163 (2001), 137–205.

• P.B., Simply-laced isomonodromy systems, Publ. Math. I.H.E.S. 116 (2012), no. 1, 1–68.

• P.B. and D. Yamakawa, Diagrams for nonabelian Hodge spaces on the affine line, C. R. Math. Acad. Sci. Paris 358 (2020), no. 1, 59–65.

• J. Douçot, Diagrams and irregular connections on the Riemann sphere, arXiv:2107.02516, 2021

https://webusers.imj-prg.fr/~philip.boalch/

Nijenhuis Geometry: symmetries, conservation laws and applications to geodesically equivalent metrics.

Alexey Bolsinov

(Loughborough)

The talk is devoted to applications of Nijenhuis operators to integrable systems of two kinds, geodesic flows of some special type of Riemannian metrics and quasilinear systems of PDEs of hydrodynamic types. These results are based on simple, but very interesting properties of the so-called gl-regular Nijenhuis operators (`gl-regularity' in the context of Nijenhuis geometry is similar to `non-degeneracy' in the context of Poisson geometry).

A Tale of Two Polytopes related to geodesic flows on spheres

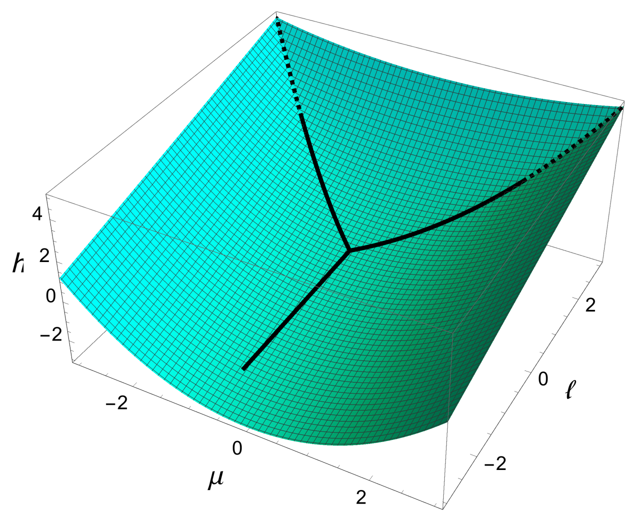
Holger R. Dullin, University of Sydney
joint work with Sean Dawson, Diana Nguyen, University of Sydney

Separation of variables for the geodesic flows on round spheres leads to a large family of integrable systems whose integrals are defined through the separation constants. Reduction by the periodic geodesic flow leads to integrable systems on Grassmanians. Specifically for the geodesic flow on the round S^3 the reduced system defines a family of integrable systems on $S^2 \times S^2$. We show that the image of these systems under a continuous momentum map defined through the action variables has a triangle as its image. The image is rigid and does not change when the integrable system is changed within the family. Each member of the family can be identified with a point inside a Stasheff polytope. Corners of the polytope correspond to toric systems (possibly with degenerations), edges correspond to semi-toric systems (in various meanings of the word), and the face corresponds to "generic" integrable systems. The momentum map is continuous but not smooth along the images of hyperbolic singularities. The corresponding quantum problem and generalisations to higher dimensional spheres will be discussed.

Bifurcations and Monodromy of the Axially Symmetric 1:1:−2 Resonance

Konstantinos Efstathiou

Zu Chongzhi Center for Mathematics and Computational Sciences
Duke Kunshan University



Abstract

In this talk, based on joint work with Heinz Hanßmann and Antonella Marchesiello [1], we consider integrable Hamiltonian systems in three degrees of freedom near an elliptic equilibrium in 1:1:−2 resonance. The integrability originates from averaging along the periodic motion of the quadratic part and an imposed rotational symmetry about the vertical axis. Introducing a detuning parameter we find a rich bifurcation diagram, containing three families of Hamiltonian Hopf bifurcations that join at the origin. I describe the monodromy of the resulting ramified 3–torus bundle as variation of the detuning parameter lets the system pass through the 1:1:−2 resonance.

References

[1] K. Efstathiou, H. Hanßmann, and A. Marchesiello. "Bifurcations and Monodromy of the Axially Symmetric 1:1:-2 Resonance". In: *Journal of Geometry and Physics* 146 (2019), p. 103493. doi: `10.1016/j.geomphys.2019.103493`.

Geometric reduction of spin trigonometric RS systems

Maxime Fairon

Laboratoire de mathématiques d'Orsay
Université Paris-Saclay, F-91405 Orsay, France
e-mail: maxime.fairon@universite-paris-saclay.fr

Abstract

The Ruijsenaars-Schneider (RS) system is a celebrated integrable system that describes the motion in a one-dimensional space of interacting particles which are invariant under Poincaré transformations. In 1995, it was realised by Krichever and Zabrodin that the system admits a spin generalisation, where the particles are endowed with internal 'spin' degrees of freedom. My goal is to review the story of the quasi-Hamiltonian reduction of such systems in the presence of a trigonometric potential. This will be based on a joint work with O. Chalykh in the complex case [1] and a recent investigation with L. Fehér in the real case [2]. As a motivation, I will start by recalling the geometric picture behind the simpler rational Calogero-Moser system with and without spin variables.

References

[1] O. Chalykh and M. Fairon, *On the Hamiltonian formulation of the trigonometric spin Ruijsenaars–Schneider system*. Lett. Math. Phys. **110** (2020) 2893-2940; arXiv:1811.08727

[2] M. Fairon and L. Fehér, *Integrable multi-Hamiltonian systems from reduction of an extended quasi-Poisson double of* U(n), Preprint (2023); arXiv:2302.14392

Poisson–Lie analogues of spin Sutherland models

László Fehér[a,b]

[a]Department of Theoretical Physics, University of Szeged
Tisza Lajos krt 84-86, H-6720 Szeged, Hungary
e-mail: lfeher@physx.u-szeged.hu

[b]Institute for Particle and Nuclear Physics
Wigner Research Centre for Physics
H-1525 Budapest, P.O.B. 49, Hungary

Abstract

We report on generalizations of those spin Sutherland models that descend by Poisson reduction from the Hamiltonian systems defined by the kinetic energy of the bi-invariant Riemannian metric on a compact semi-simple Lie group. The systems of interest are obtained by reducing the 'master integrable systems' on Heisenberg doubles representing the natural Poisson–Lie counterparts of the systems of free motion modeled on the cotangent bundles. We will demonstrate the degenerate integrability of the reduced systems after restriction to a dense open part of the Poisson quotient. The pertinent subset of the reduced phase space corresponds to restriction to the principal orbit type regarding the G-actions both on the Heisenberg double and on a dense open subset of the 'space of constants of motion'. Degenerate integrability on the generic symplectic leaves of the restricted Poisson quotient will also be proven. In addition, two characterizations of the reduced Poisson structure will be presented, the first one incorporates a dynamical r-matrix, while the second one utilizes variables consisting of canonical pairs and decoupled generalized spin variables underlying the interpretation of the reduced systems as Ruijsenaars–Schneider type deformations of the spin Sutherland models.

Similar results on the integrable systems that result from reductions of the internally fused quasi-Poisson doubles will be outlined at the end of the talk.

References

[1] L. Fehér, *Poisson–Lie analogues of spin Sutherland models*, Nucl. Phys. B 949, 114807 (2019); arXiv:1809.01529

[2] L. Fehér, *Poisson reductions of master integrable systems on doubles of compact Lie groups*, Ann. Henri Poincaré 24, 1823-1876 (2023); arXiv:2208.03728

[3] L. Fehér, *Poisson–Lie analogues of spin Sutherland models revisited*, in preparation

Poisson geometry of planar networks and simple Lie groups

Anton Izosimov,
University of Arizona

Elements of the matrix group $GL(n)$ can be encoded by weighted planar networks (directed graphs with numbers written on edges) with n sources and n sinks. Such graphical representation is useful for studying matrix factorizations, totally positive matrices etc. Furthermore, Gekhtman, Shapiro, and Vainshtein showed that such networks also capture Poisson geometry of $GL(n)$ endowed with a standard multiplicative Poisson bracket. In the talk I will present a similar graphical representation of standard Poisson structures on simple Lie groups of type B and C.

Families of (hyper)semitoric systems

Yohann Le Floch

IRMA, Université de Strasbourg

A semitoric system is a four-dimensional integrable system whose momentum map has a component generating an effective S^1-action and possesses mild singularities. These systems have been classified a few years ago by Pelayo and Vũ Ngọc (for the generic case) and later by Palmer, Pelayo and Tang (for the fully general case), thanks to several invariants; however, the construction of a system with given invariants involves symplectic gluing of some local normal forms. My goal will be to review these notions and to explain some attempts to answer the following broad question: can one construct a fully explicit (i.e. whose momentum map is given by global explicit formulas) semitoric system with a given partial list of invariants in a relatively simple way? I will discuss some results that this seemingly naive question led to, and in particular I will mention more general systems with S^1-symmetry called hypersemitoric systems. This is based on past and ongoing work with Joseph Palmer (University of Illinois Urbana-Champaign).

Integrable systems on nilpotent Lie subalgebras via cluster theory

Yanpeng Li

(Sichuan University)

Let G be a complex semisimple Lie group and Lie(G) = $\mathfrak{n} \oplus \mathfrak{h} \oplus \mathfrak{n}_-$ be a triangular decomposition. We construct an integrable system on the dual space of $\mathfrak{n}_-$ equipped with the Kirillov-Kostant-Souriau Poisson structure, using a Poisson-cluster seed for G/B (equipped with the standard Poisson structure) arising from symmetric Poisson CGL extension. This is joint work in progress with Yu Li and Jiang-Hua Lu.

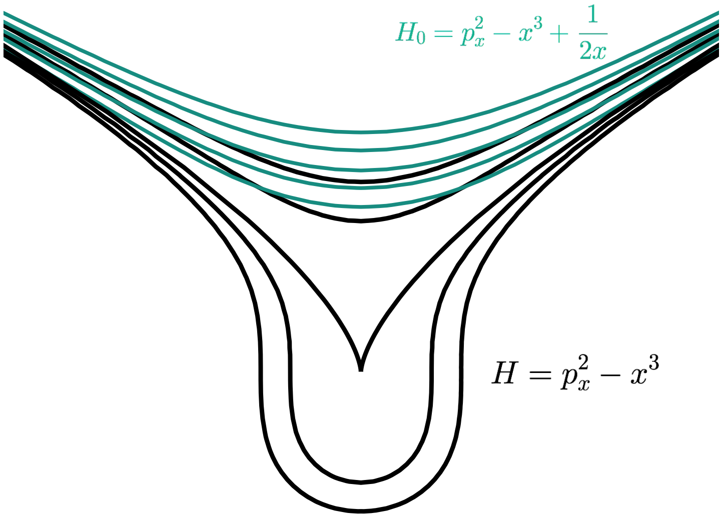
NEW TOPOLOGICAL INVARIANTS OF SCATTERING OF LIOUVILLE INTEGRABLE SYSTEMS

NIKOLAY N. MARTYNCHUK

Bernoulli Institute for Mathematics, Computer Science and AI, University of Groningen

Scattering theory first entered the topological theory of Liouville integrable systems in the works [1,3], where the notion of (quantum) scattering monodromy was introduced (in the context of planar potential scattering with a rotationally-symmetric potential). Subsequently these works were generalised in [5] to include integrable systems with many degrees of freedom that are not necessarily invariant under a circle/torus action; see also [4].

In this talk, we shall show how similar ideas lead to what we call *scattering Fomenko-Zieschang invariants* (or *scattering marked molecules*); see [2] for their description in the compact case. These invariants can, in principle, be computed for all Liouville integrable systems with invariant cylinders, as long as an appopriate scattering pair can be defined. Several explicit examples, including the case of an A_2 singularity, will be discussed.



References

[1] L. Bates and R. Cushman. Scattering monodromy and the A1 singularity. *Central European Journal of Mathematics*, 5(3):429–451, 2007.

[2] A. V. Bolsinov and A. T. Fomenko. *Integrable Hamiltonian Systems: Geometry, Topology, Classification*. CRC Press, 2004.

[3] H. R. Dullin and H. Waalkens. Nonuniqueness of the phase shift in central scattering due to monodromy. *Physical Review Letters*, 101:070405, 2008.

[4] K. Efstathiou, A. Giacobbe, P. Mardešić, and D. Sugny. Rotation forms and local Hamiltonian monodromy. *Journal of Mathematical Physics*, 58(2):022902, 2017.

[5] N. Martynchuk, H. Dullin, K. Efstathiou, and H. Waalkens. Scattering invariants in Euler's two-center problem. *Nonlinearity*, 32(4):1296–1326, 2019.

Speaker: Eckhard Meinrenken, University of Toronto

Title: Moduli spaces of hyperbolic 0-metrics

Abstract: A hyperbolic 0-metrics on a compact surfaces Σ with boundary $\partial\Sigma$ is a hyperbolic metric on the interior, with a boundary behaviour similar to that of the Poincaré metric on the upper half plane. We define the Teichmüller or Riemann moduli spaces of such metrics as quotients

$$\mathrm{Teich}(\Sigma) = \mathrm{Hyp}(\Sigma)/\operatorname{Diff}_0(\Sigma, \partial\Sigma), \quad \mathcal{M}(\Sigma) = \mathrm{Hyp}(\Sigma)/\operatorname{Diff}^+(\Sigma, \partial\Sigma),$$

where $\operatorname{Diff}^+(\Sigma, \partial\Sigma)$ are orientation preserving diffeomorphisms fixing the boundary, and $\operatorname{Diff}_0(\Sigma)$ is the identity component. We show that these spaces have natural symplectic structures, and are examples of Hamiltonian Virasoro spaces. Bases on joint wok with Anton Alekseev.

NONEQUILIBRIUM THERMODYNAMICS AS A SYMPLECTO-CONTACT REDUCTION AND RELATIVE INFORMATION ENTROPY

YONG-GEUN OH

ABSTRACT. Starting from Carathedory and Hermann, contact geometry is proposed as the correct geometric framework of thermodynamics, especially of its equilibrium thermodynamics. It has been observed in this formulation that the state of thermodynamic equilibrium can be interpreted as a Legendrian submanifold in thermodynamic phase space (TPS). In this lecture, we will explain the origin of aforementioned contact structure in TPS, in particular of its odd dimensionality, as a symplecto-contact reduction of the space of the probability distribution space of the statistical phase space (SPS) of many-body systems, which we call the kinetic theory phase space (KTPS). The relative information entropy associated to the probability distribution will be emphasized in this derivation: It plays the role of a generating function of thermodynamic equilbria as a Legendrian submanifold, after a collective symplectic reduction. This Legendrian submanifold is not necessarily graph-like. We then interpret the Maxwell construction in thermodynamics as the procedure of finding a continuous, not necessarily differentiable, thermodynamic potential and explain the associated phase transition. We identify the Maxwell construction with the algorithm of finding a graph selector in symplecto-contact geometry and in the Aubry-Mather theory of dynamical system. (This is based on a joint work with my student Jinwook Lim.)

CENTER FOR GEOMETRY AND PHYSICS, INSTITUTE FOR BASIC SCIENCE (IBS) & POSTECH, KOREA
Email address: yongoh1@postech.ac.kr

Lifting Hamiltonian torus actions to integrable systems

Joseph Palmer

University of Illinois at Urbana-Champaign and University of Antwerp

An *integrable system* consists of a $2n$-dimensional symplectic manifold (M, ω) equipped with a set of n real-valued functions which are independent and Poisson commute. In the case that the Hamiltonian flow of these functions generates an effective action of the n-torus, the system is called *toric*. Toric integrable systems are well-understood thanks to results of Atiyah, Guillemin-Sternberg, and Delzant which classify them in terms of a certain class of polytopes.

Similarly, a *complexity-one space* is the Hamiltonian action of an $(n-1)$-torus, so, roughly, it can be thought of as a set of $n-1$ independent, Poisson commuting functions. In particular, a complexity-one space on a 4-manifold is a Hamiltonian S^1-action. This talk is concerned with various versions of the following question: given a complexity-one space generated by functions $g_1, \ldots, g_{n-1}$, when can one additional function f be found so that $(g_1, \ldots, g_{n-1}, f)$ forms an integrable system of a certain desirable type?

We will start with the case that $n = 2$ by reviewing a result by Karshon about which S^1-actions can be extended to a toric system on a 4-manifold, and then explaining a recent result, that I obtained jointly with S. Hohloch, about a class of integrable systems to which all S^1-actions can be lifted. I will also explain recent results, joint with S. Tolman and Y. Liu, about lifting complexity-one spaces to toric systems in higher dimensions, generalizing Karshon's result.

Complete integrability of the subriemannian geodesic flows over the seven-dimensional sphere

Daisuke Tarama*

This talk deals with the complete integrability of the subriemannian geodesic flows over the 7-dimensional sphere. Besides the well-known contact and quaternionic Hopf distributions, trivializable (bracket-generating) subriemannian structures of different ranks are given by means of Clifford representations. The geodesic flows are formulated as Hamiltonian systems on the cotangent bundle to the sphere and their complete integrability is discussed on the basis of Thimm's method proposed around 1980. The talk is based on the collaborations with Wolfram Bauer and Abdellah Laaroussi (Leibniz Univ. Hannover).

*Department of Mathematical Sciences, Ritsumeikan University. 1-1-1 Nojihigashi, Kusatsu, Shiga, 525-8577, Japan. Partially supported by JSPS Kakenhi Grant Number JP19K14540,JP22H01138, JP23H04481.

COMPLEXITY ONE NONDEGENERATE INTEGRABLE SYSTEMS

SUSAN TOLMAN
UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN

Let (M, ω) be a compact, connnected $2n$-dimensional symplectic manifold. Fix $f \colon M \to \mathbb{R}^n$ such that (M, ω, f) is an integrable system. If M is four-dimensional then we say that (M, ω, f) is *semitoric* if (1) every singular point is non-degenerate without hyperbolic blocks, and (2) the first component of f generates a circle action. Semitoric systems have been extensively studied and have many nice properties: for example, the preimages $f^{-1}(x)$ are all connected. Unfortunately, although there are many interesting examples of semitoric systems, the class has some limitation. For example, there are blowups of $S^2 \times S^2$ with Hamiltonian circle actions which cannot be extended to semitoric systems. We expand the class of semitoric systems in two ways. First, we allow symplectic manifolds of arbitrary dimension, as long as the first $n-1$ coordinates of generate an $n-1$-dimensional "complexity one" torus action. Second, we allow all non-degenerate singularities, including those with hyperbolic block. Although, in this larger class, it's no longer true that the preimage $f^{-1}(x)$ is always connected, D. Sepe and I can prove, For example, if each preimage contains at most one singular point then the preimages are all connected iff no singular point with a connected torus stabilizer has contains a hyperbolic block. I will also discuss work in progress with J. Palmer, where we study which complexity one torus actions can be extended to nondegenerate integrable systems.

Date: May 2023.

Cornelia Vizman (West University of Timisoara)

Title:
Universal central extensions of Lie algebras of vector fields

with Bas Janssens (Delft University of Technology) and Leonid Ryvkin (University Claude Bernard Lyon 1)

Abstract:
I will start by reminding an older work [1] on the universal central extension of the Lie algebra of Hamiltonian vector fields, then I will explain our new work on the universal central extension of the Lie algebra of exact divergence free vector fields. Both results were conjectured in [2]. Unlike the Hamiltonian setting, a detour in the realm of Leibniz algebras is needed in the divergence free setting.

References:
[1] B. Janssens, C. Vizman, Universal central extension of the Lie algebra of Hamiltonian vector fields, IMRN, 2016.16(2016) 4996-5047.
[2] C. Roger, Extensions centrales d'algebres et de groupes de Lie de dimension infinie, algèbre de Virasoro et généralisations, Rep. Math. Phys., 35(1995) 225–266.

Symplectic A_k singularities, Schrödinger operators, and inverse spectral theory

San Vũ Ngọc

Université de Rennes

Joint work with Nikolay Martynchuk.

I will first survey classical results on symplectic normal forms for Morse Hamiltonians in $\mathbb{R}^2$, together with the corresponding microlocal normal forms for pseudodifferential operators. These results have been used with some success for the (semiclassical) inverse spectral problem: can you recover the principal symbol from the quantum spectrum?

I will then turn to the recent studies of A_k singularities, which are a natural family of degenerate singularities. On the quantum side, they give rise to interesting normal forms involving Schrödinger operators, which motivates new conjectures concerning the inverse spectral problem.

Toric geometry with elliptic singularities

Aldo Witte

June 2023

Motivated by the study of generalized complex structures, elliptic symplectic structures were introduced by Cavalcanti and Gualtieri. These are symplectic forms with singularities along codimension-two submanifolds. They fit in the larger class of symplectic structures on Lie algebroids, which have received considerable attention in the past decade. Elliptic symplectic structures are supported by singular fibrations, which are surprisingly the same types of maps as moment maps in (semi)toric geometry. In this talk I will study elliptic symplectic structures for which this fibration is induced by a torus action and has Lagrangian fibres. This gives rise to a notion of toric symplectic structures with elliptic singularities, similar to toric symplectic structures with logarithmic singularities as studied by Gualtieri-Li-Pelayo-Ratiu. Based on joint work with Gil Cavalcanti and Ralph Klaasse.