COMPLEXITY ONE NONDEGENERATE INTEGRABLE SYSTEMS

SUSAN TOLMAN
UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN

Let (M, ω) be a compact, connnected $2n$-dimensional symplectic manifold. Fix $f\colon M \to \mathbb{R}^n$ such that (M, ω, f) is an integrable system. If M is four-dimensional then we say that (M, ω, f) is *semitoric* if (1) every singular point is non-degenerate without hyperbolic blocks, and (2) the first component of f generates a circle action. Semitoric systems have been extensively studied and have many nice properties: for example, the preimages $f^{-1}(x)$ are all connected. Unfortunately, although there are many interesting examples of semitoric systems, the class has some limitation. For example, there are blowups of $S^2 \times S^2$ with Hamiltonian circle actions which cannot be extended to semitoric systems. We expand the class of semitoric systems in two ways. First, we allow symplectic manifolds of arbitrary dimension, as long as the first $n-1$ coordinates of generate an $n-1$-dimensional "complexity one" torus action. Second, we allow all non-degenerate singularities, including those with hyperbolic block. Although, in this larger class, it's no longer true that the preimage $f^{-1}(x)$ is always connected, D. Sepe and I can prove, For example, if each preimage contains at most one singular point then the preimages are all connected iff no singular point with a connected torus stabilizer has contains a hyperbolic block. I will also discuss work in progress with J. Palmer, where we study which complexity one torus actions can be extended to nondegenerate integrable systems.

Date: May 2023.