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Vortex dynamics and symplectic Dirac operators

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The CCR (canonical commuting relation) and CAR algebras (canonical anticommuting relation) are fundamental algebras in theoretical physics used for the study of bosons and fermions. From a mathematical viewpoint, these algebras are named the Weyl algebra (or symplectic Clifford algebra) and Clifford algebra. The Clifford algebra is constructed on a vector space equipped with a symmetric bilinear form, whereas the Weyl algebra requires an even dimensional vector space equipped with a skew-symmetric bilinear form (or symplectic form). Using the generators of the Clifford (resp. Weyl) algebra, one can associate a natural first order spin (resp. metaplectic) invariant differential operator by contracting the Clifford algebra elements using the bilinear form (resp. the symplectic form) with derivatives. The theory which studies the solutions of the Dirac operator is known as Clifford analysis and can be seen as a hypercomplex function theory. In the first part of the thesis, we will study the symplectic Dirac operator, from an orthogonal point of view. In addition, we provide the foundations of what we will call a 'hermitian variant' of symplectic Clifford analysis, where we incorporate the additional datum of a compatible complex structure and study the associated solution space using algebraic techniques and arrive at a Fischer decomposition.

In the second part of the thesis, we provide tools to study the dynamics of point vortex dynamics on the complex projective spaces and the six-dimensional flag manifold. These are the only Kähler twistor spaces arising from 4-manifolds. We give an explicit expression for Green's function on the projective space which enables us to determine the Hamiltonian (the energy of the system) and the equations of motions for the point vortex problem. Moreover, we determine the momentum map on the flag manifold, which is a key ingredient in understanding the dynamics better.