

Quasi-rigidity in Linear Dynamics

Antoni López-Martínez
Universitat Politècnica de València

Linear Dynamics studies the orbits generated by the iterations of a continuous linear operator $T : X \rightarrow X$ acting on a (usually infinite-dimensional) topological vector space X . During the last 40 years, the two most studied properties in this branch of mathematics have been hypercyclicity and chaos. However, the notion of *recurrence* has recently been considered in the linear context (see the 2014 paper [2]), generating a new line of research which is full of natural questions and open problems.

In this talk we will focus on the so-called $T \oplus T$ -recurrence problem originally posed in [2, Question 9.6]:

– If T is a recurrent operator, does it follow that $T \oplus T$ is also a recurrent operator?

In order to give a complete answer to this question, we will introduce the notion of *quasi-rigidity* (which will be the “recurrence version” of the weak-mixing property) and then we will construct a recurrent but not quasi-rigid operator. We will end the talk by discussing how to use quasi-rigidity to study the linear structure of the set of recurrent vectors (lineability, dense lineability and spaceability).

The talk is based on [4, 5, 6, 7]. In particular, [4, 5] are a joint works with Sophie Grivaux and Alfred Peris, and [7] is a joint work with Quentin Menet.

References

- [1] J.-M. Augé. Linear Operators with Wild Dynamics. *Proc. Amer. Math. Soc.*, **140**(6) (2012), 2103–2116.
- [2] G. Costakis, A. Manoussos, and I. Parissis. Recurrent linear operators. *Complex Anal. Oper. Theory*, **8**(2014), 1601–1643.
- [3] M. De La Rosa and C. Read. A hypercyclic operator whose direct sum $T \oplus T$ is not hypercyclic. *J. Oper. Theory*, **61** (2009), 369–380.
- [4] S. Grivaux, A. López-Martínez, and A. Peris. Questions in linear recurrence I: The $T \oplus T$ -recurrence problem. *Anal. Math. Phys.*, **15**(1) (2025), 26 pages.
- [5] S. Grivaux, A. López-Martínez, and A. Peris. Questions in linear recurrence II: Lineability properties. *Banach J. Math. Anal.*, **19**(61) (2025), 28 pages.
- [6] A. López-Martínez. Recurrent subspaces in Banach spaces. *Int. Math. Res. Not. IMRN*, **2024**(11) (2024), 9067–9087.
- [7] A. López-Martínez and Q. Menet. Two remarks on the set of recurrent vectors. *J. Math. Anal. Appl.*, **541**(1) (2025), 17 pages.