

Hamiltonian Group Actions and Fixed Points

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Symmetry plays a central role in symplectic geometry and momentum maps are a powerful tool for studying it. This minicourse will introduce Hamiltonian actions of compact Lie groups on symplectic manifolds, with particular focus on actions of tori and the significance of their fixed points. Beginning with the momentum map and its basic properties, we will explore how the topology of fixed point components together with local isotropy data can be used to assemble global topological and geometric features of the ambient manifold. We will examine localization techniques, convexity theorems, and fixed-point formulas that use symmetries and their fixed points to compute invariants such as (equivariant) cohomology and (equivariant) characteristic classes. Along the way, we will discuss numerous examples, including toric manifolds, coadjoint orbits, and moduli spaces. Depending on time and audience interest, we may also discuss recent developments and open questions concerning Hamiltonian torus actions, equivariant topology, and the classification of symplectic manifolds with large symmetry groups. The course is intended for graduate students with some background in differential geometry and aims to give a broad overview grounded in concrete examples.