The solutions and monodromy of the irregular Knizhnik–Zamolodchikov equations

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In this talk, we study the *Knizhnik–Zamolodchikov (KZ) equations* which is a system of 1^{st} order linear PDEs associated to a connection with regular singular points (i.e. simple poles). They are not only fundamental to our understanding of physics (e.g. conformal field theory and statistical mechanics), but also in our understanding of pure mathematics. In particular, they have been used to study the representation theory of affine Lie algebras and quantum groups, knot theory and hyperplane complements. Of particular interest in our talk is the *monodromy* (i.e. parallel transport, analytic continuation) of these equations. In the late 1980s, T. Kohno and V. G. Drinfeld established a theorem which states that the monodromy representation of the KZ equations is equivalent to a representation obtained from a certain *'quantum group'*. This leads to a surprising link between two supposedly unrelated areas of mathematics, namely differential geometry and algebra.

We also discuss the recently introduced *irregular KZ equations*, which is a generalisation of the KZ equations where we eliminate the requirement that the singular points of the associated connection are regular. Such equations are useful in the study of deformations of certain Hamiltonian systems (e.g. *isomonodromy systems*). We compute solutions to these equations and compute the monodromy explicitly in the case that the chosen Lie algebra is $g = sl_2(C)$. Finally, we discuss the significance of these results and state some future research prospects.