Hamiltonian group actions and fixed points

Mantra: act globally, compute locally

Poisson 2026
Summer School
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Cornell University

Outline - Lectures 1 & 2

- Symplectic geometry (with lots of examples)
- Group actions & fixed points (with lots of examples)
- Localization (with lots of examples)
- Symplectic reduction (how to take a quotient)
  (with lots of examples)

Outline - Lectures 3 & 4

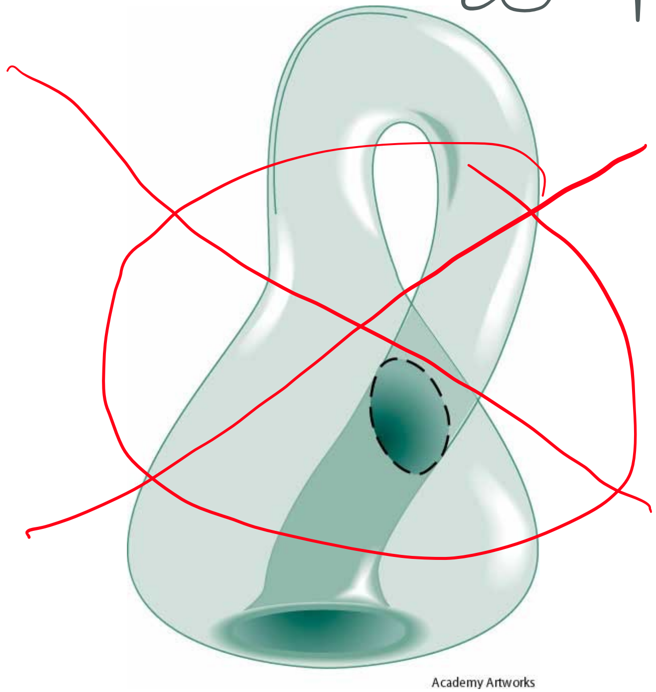
- Classification results: toric, complexity one
- How much does cohomology convey?
- Symplectic embeddings (if time)

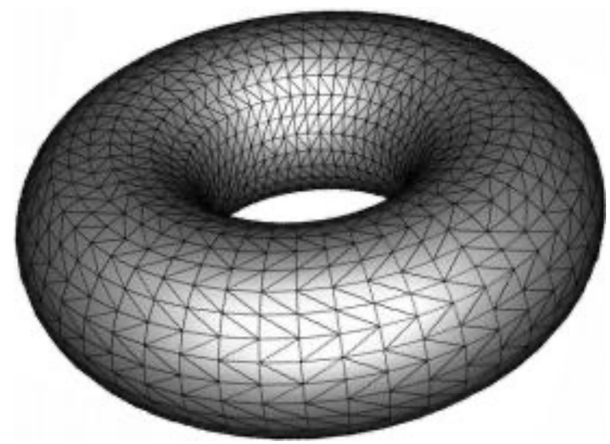
Symplectic manifolds

A symplectic manifold is a manifold with a 2-form $\omega \in \Omega^2(M)$ that is

- closed
- non-degenerate

$\omega^{top} = \mathrm{Vol}$

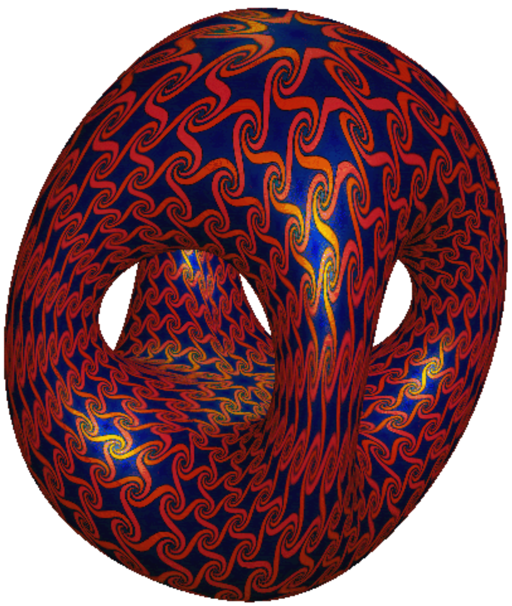
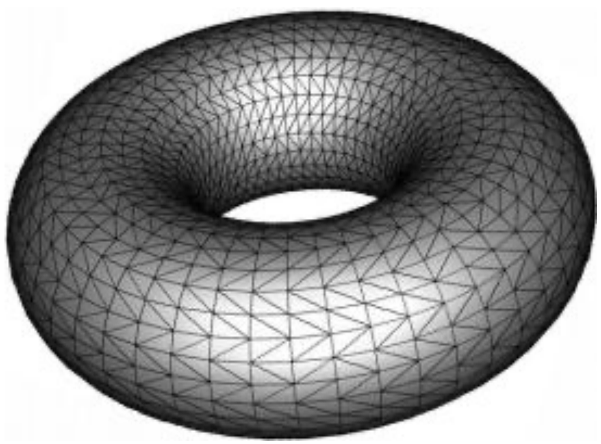




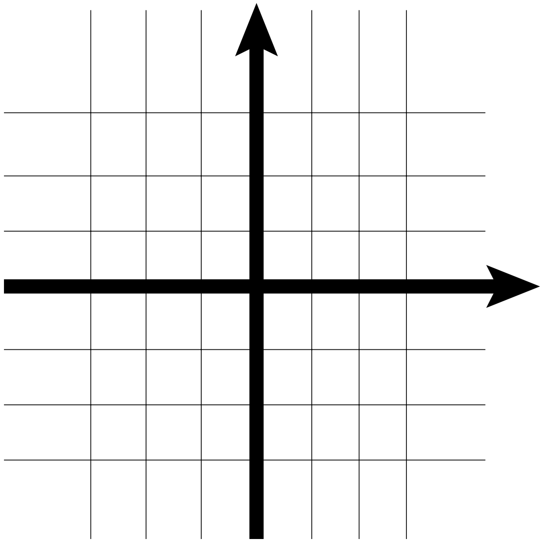
Symplectic manifolds

A symplectic manifold is a manifold with a two-form $\omega \in \Omega^2(M)$ that is:

- Closed: $d\omega = 0$
- Non-degenerate: $\omega^n = d\mathrm{Vol} \rightsquigarrow$ M is $2n$-dimensional & orientable

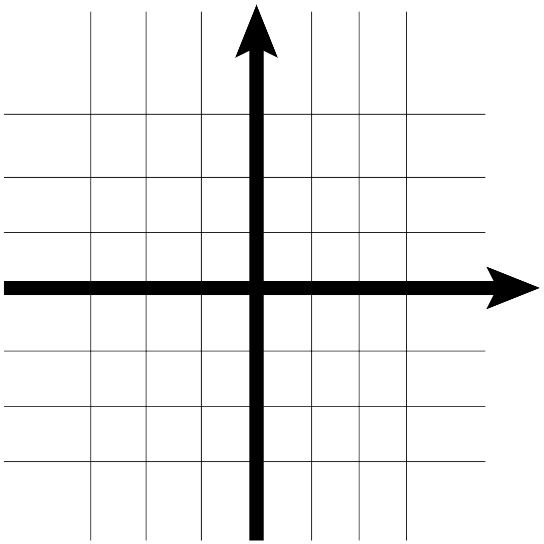




Symplectic manifolds



$$(\mathbb{R}^2, \omega = dx \wedge dy) \rightsquigarrow \left(\mathbb{R}^{2n}, \omega = \sum dx_i \wedge dy_i\right)$$

Symplectic manifolds



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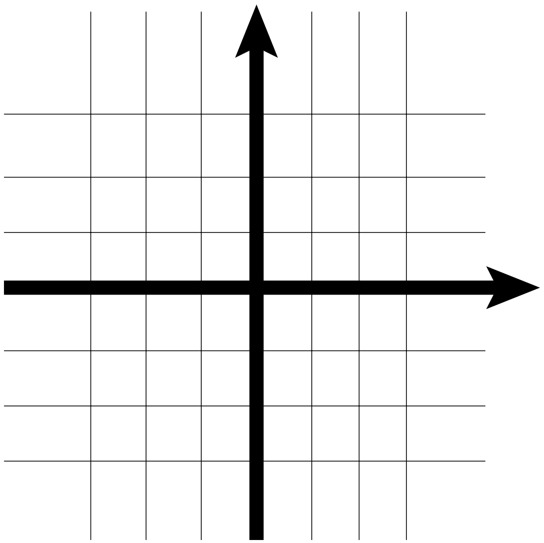
Darboux's Theorem:

We may always choose coordinates $x_1, \ldots, x_n, y_1, \ldots, y_n$ on M so that locally

$$\omega = \sum dx_i \wedge dy_i.$$

Symplectic manifolds



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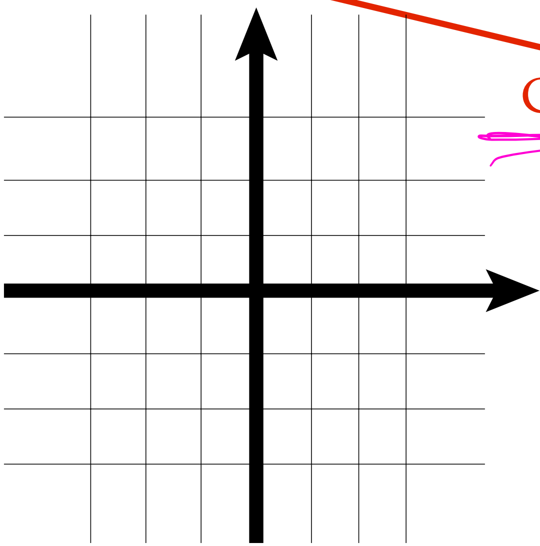
on chart

$$\omega = \sum dx_i \wedge dy_i.$$

Instead: how big can you make

$\rightsquigarrow$ There are no local invariants (like curvature).

Symplectic manifolds



Calque of "complex" introduced by Weyl (1939)

Complex: Latin *com-plexus* "braided together"

Symplectic: Greek *συμ - πλεκτικός*

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Compact examples

- Even-dimensional spheres $S^{2n} = \{ \vec{x} \in \mathbb{R}^{2n+1} \mid \sum x_i^2 = 1 \}$

area

Higher dim: doesn't work because Stoke's Thm

$\omega = d\alpha$

$$\int_M \omega^n = \int_M d\alpha(\wedge\omega^{n-1}) = \int_{\partial M} \alpha \wedge \omega^{n-1}$$

$\Rightarrow$ $[\omega] \neq 0 \in H^2(M)$ for M compact

Compact examples

$(n \geq 2)$

- ~~Even-dimensional spheres~~ $\cancel{S^{2n} = \{\vec{x} \subset \mathbb{R}^{2n+1} \mid \sum x_i^2 = 1\}}$
- Complex projective space $\mathbb{C}P^{n-1} = \{V \subseteq \mathbb{C}^n \mid \underline{\dim_{\mathbb{C}}(V) = 1}\}$

cell decomp

1 cell in

each even degree

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- Flag varieties

$$\mathcal{F}\ell ags(\mathbb{C}^n) = \{V_0 \subseteq V_1 \subseteq \cdots \subseteq V_n = \mathbb{C}^n \mid \dim_{\mathbb{C}}(V_i) = i\}$$

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Coadjoint orbits $\mathcal{O}_\lambda$

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symplectic leaves for Lie-Poisson structure on $\mathfrak{g}^*$ $(G = SU(n))$

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- Smooth complex projective varieties

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- Toric varieties (smooth)

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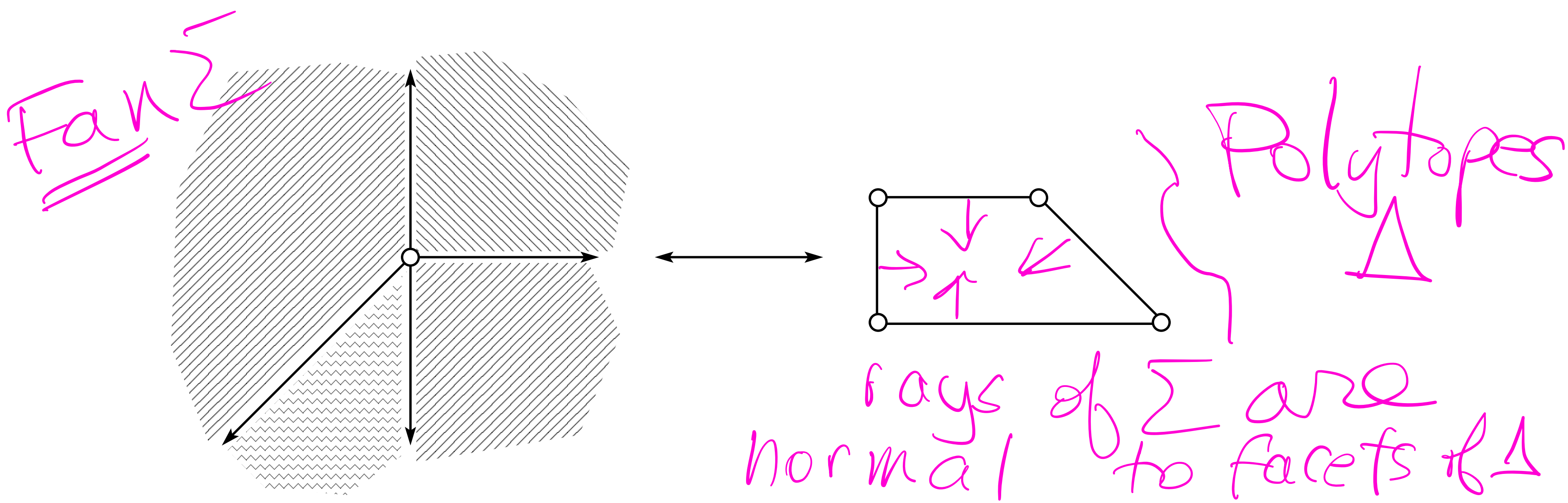

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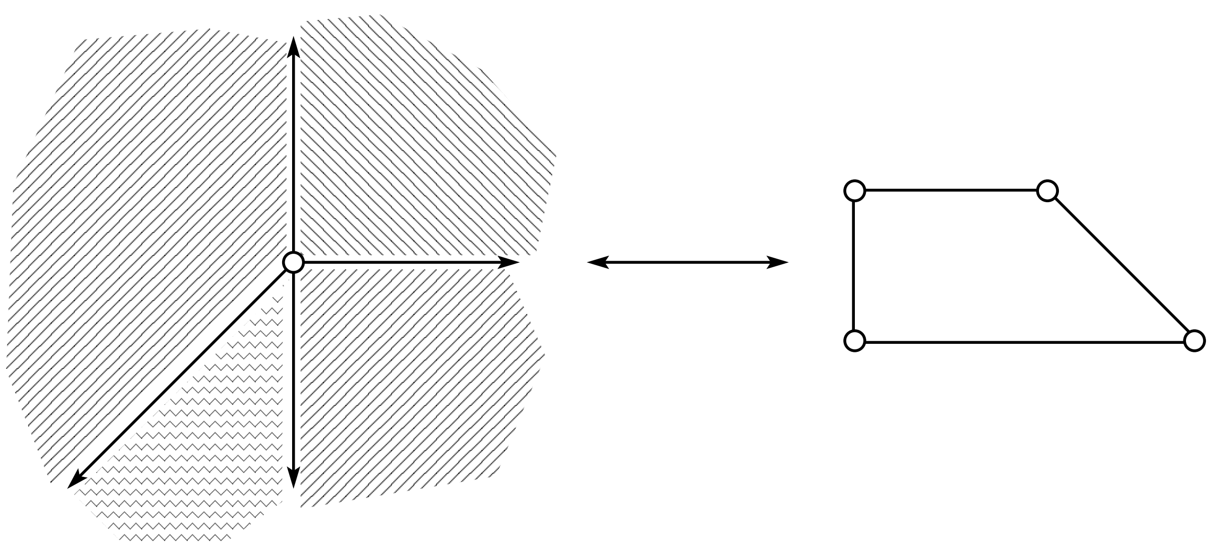
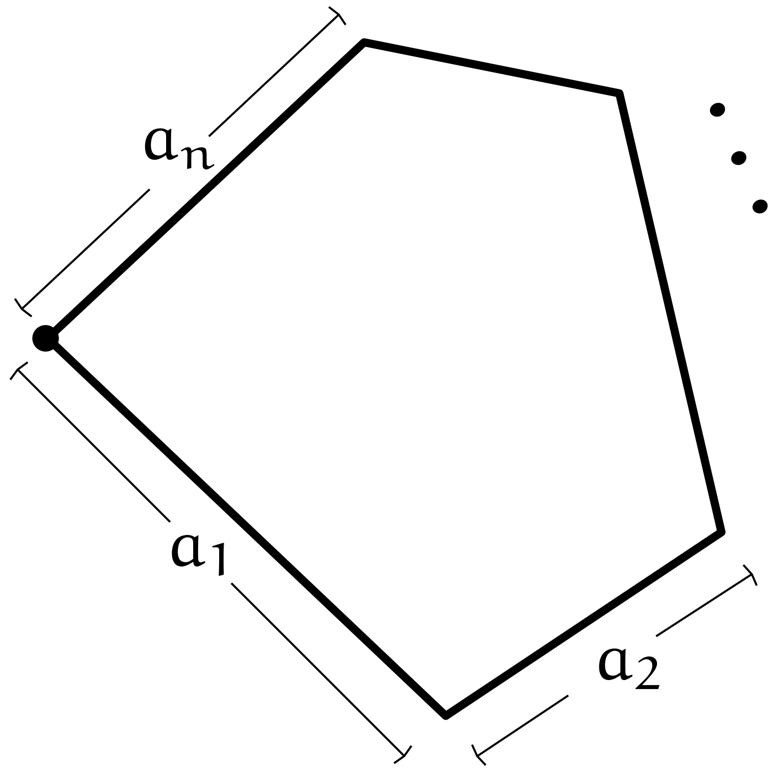
- Smooth complex projective varieties
- Toric varieties
- Based loops $\Omega G = \{\gamma : S^1 \to G \mid \gamma(\mathrm{Id}) = \mathrm{Id}\}$

G = cpt. Lie group



Example: $\mathcal{P}ol_d(a_1, \ldots, a_n)$



$\in \mathcal{P}ol_2(a_1, \ldots, a_5)$

$$\mathcal{P}ol_d(a_1, \ldots, a_n) = \frac{\left\{ (\overrightarrow{v_1}, \ldots, \overrightarrow{v_n}) \;\middle|\; \overrightarrow{v_i} \in \mathbb{R}^d,\ |v_i| = a_i,\ \sum \overrightarrow{v_i} = \overrightarrow{0} \right\}}{SO(d)}$$

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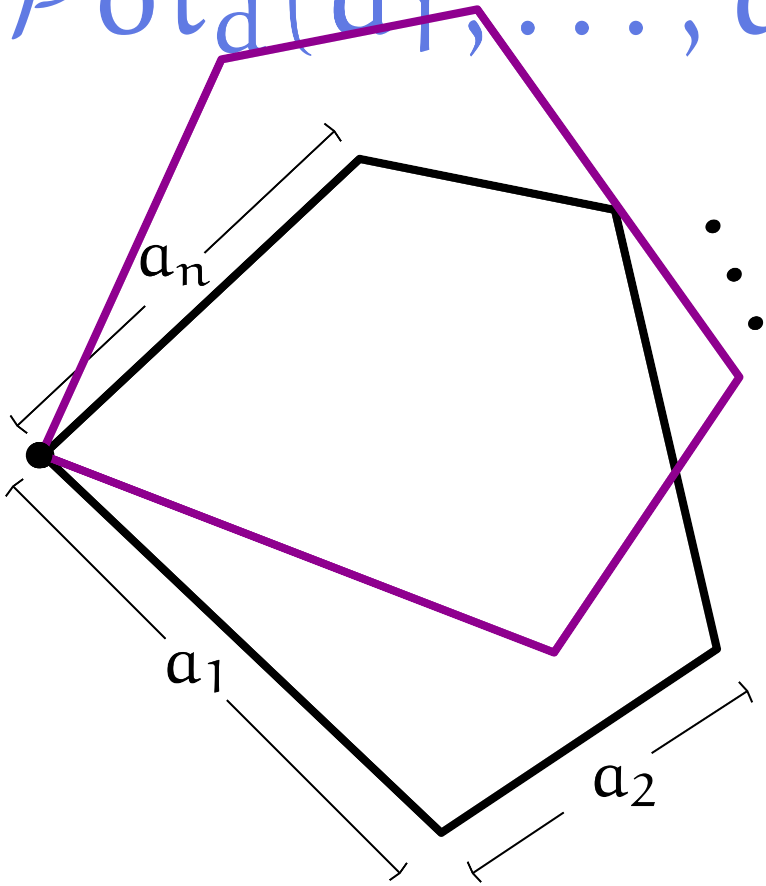

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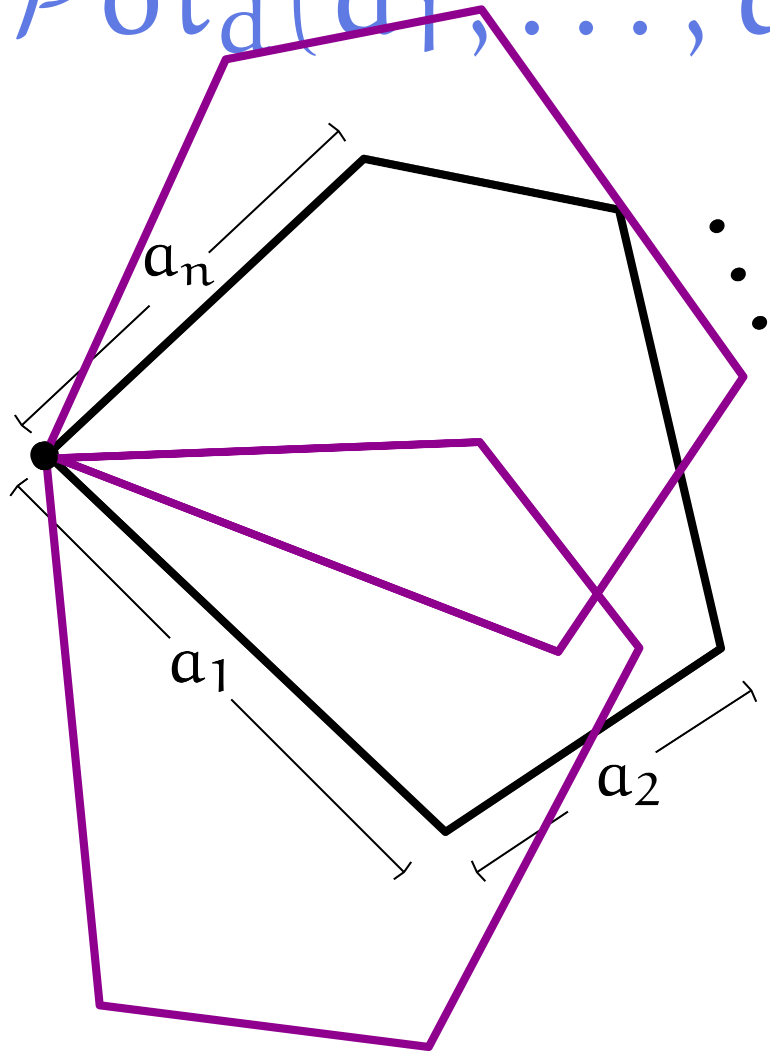

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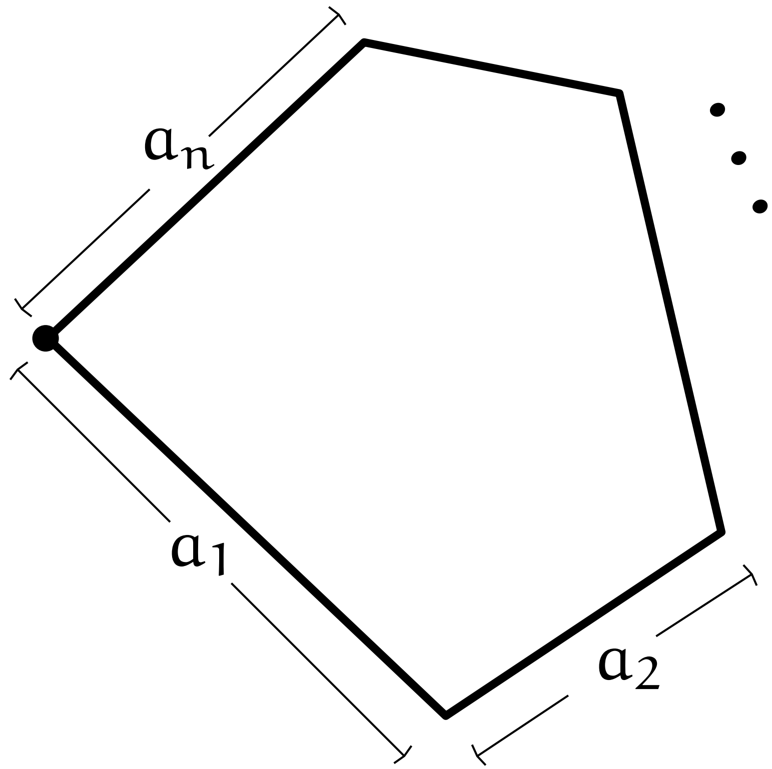

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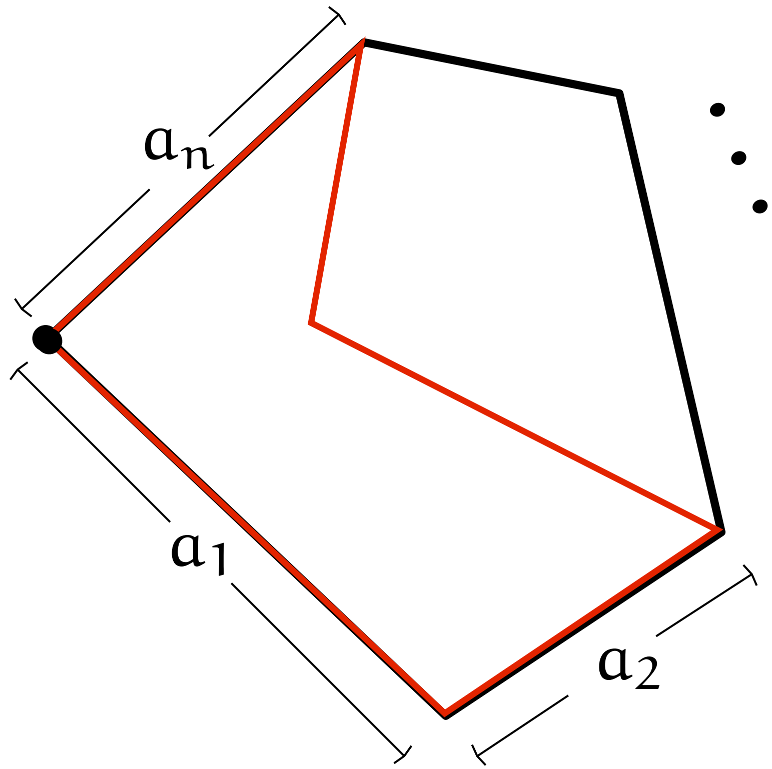
Example: $\mathcal{P}ol_d(a_1, \ldots, a_n)$ = Polygon space



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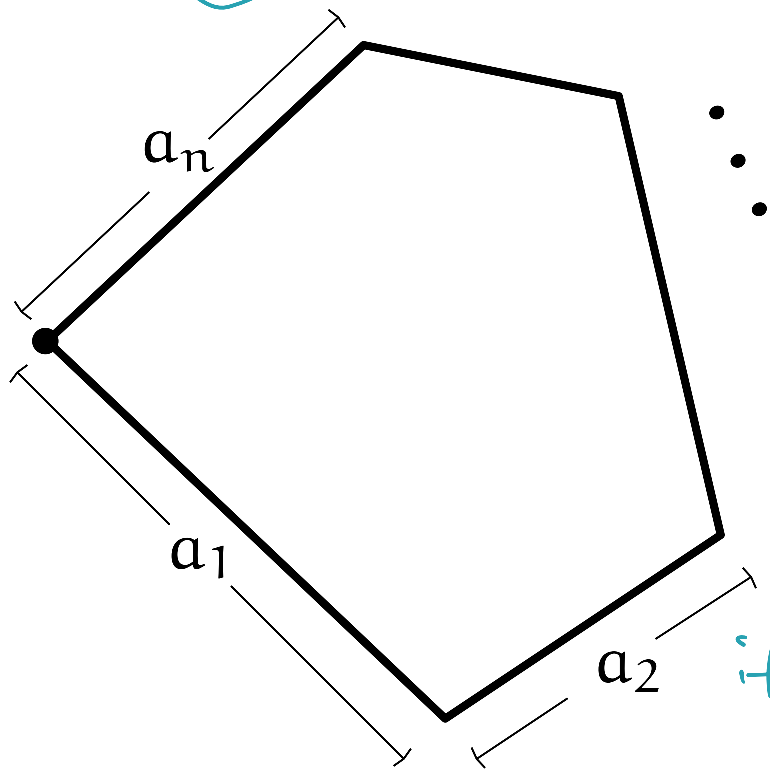


$n = 1, 2$ $\emptyset$
or pt
if $n = 2$
$a_1 = a_2$

$\in \mathcal{P}ol_2(a_1, \dots, a_5)$

$n = 3$
if Δ ineq $\Rightarrow$ can make Δ
$\Rightarrow$ pt.

$$\mathcal{P}ol_d(a_1, \dots, a_n) = \frac{\left\{(\vec{v_1}, \dots, \vec{v_n}) \;\middle|\; \vec{v_i} \in \mathbb{R}^d,\ |v_i| = a_i,\ \sum \vec{v_i} = \vec{0}\right\}}{SO(d)}$$

$d = 3$

$\mathcal{P}ol_3(a_1, \dots, a_n)$ is symplectic! N.B. d=3!!

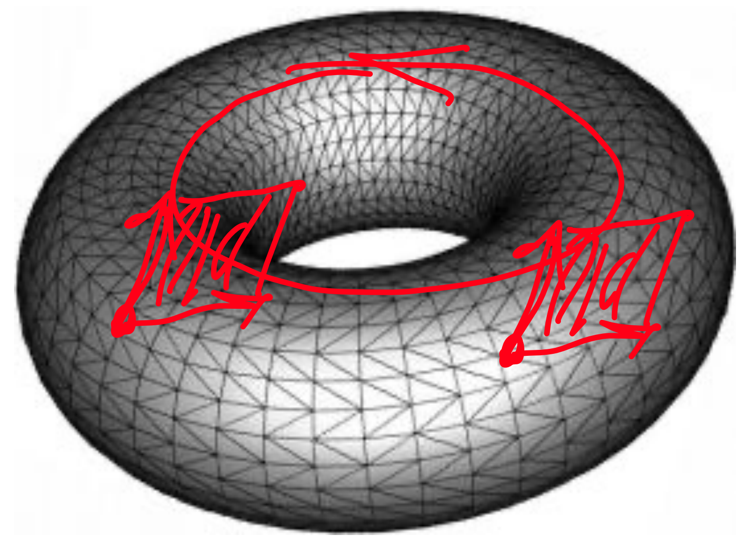
Symplectic actions

Symplectic manifolds often exhibit symmetries, encoded by a group action. (It's a hard topological question, "How many manifolds do or do not have symmetries?" ...)

DEF: A group action $G \circlearrowright M$ is **symplectic** if it preserves ω ; that is,

$$\tau_g^* \omega = \omega \quad \forall\, g \in G.$$

$S^1 \circlearrowright S^2$ by rotation



$S^1 \circlearrowright T^2$ by rotation

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$S^1 \circlearrowright S^2$ by rotation

$SO(3) \circlearrowright S^2$ by multiplication

~~$O(3) \circlearrowright S^2$ by multiplication~~

reverses orientation!

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DEF: Let G be a Lie group with Lie algebra $\mathfrak{g}$. Suppose $G \circlearrowright M$. For any $\xi \in \mathfrak{g}$, we may define a vector field on M by,

$$\mathcal{X}_\xi(p) = \frac{d}{dt}\left[\exp(t\xi)\cdot p\right]\Big|_{t=0}.$$

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$d\iota_X + \iota_X d$

Lie deriv along v.fields

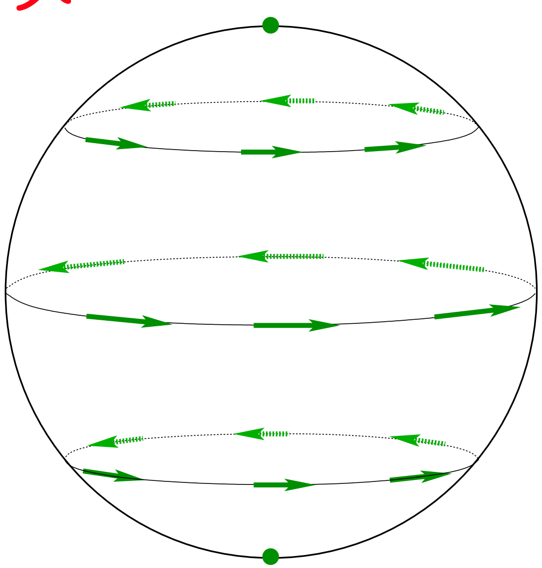
$\mathcal{L}_{X_\xi}\omega = 0$ is zero

$\|$

$$0 = d(\omega(X_\xi, -)) + \iota_{X_\xi} d\omega$$

0

1-form $\omega(X_\xi,-)$ closed.



$S^1 \circlearrowright S^2$ by rotation $\rightsquigarrow$ Vector field parallel to latitude lines

Hamiltonian actions

$$\boxed{d\Big(\omega(\mathcal{X}_\xi,\cdot)\Big) = 0}$$

DEF: Suppose $G \circlearrowright (M,\omega)$. We say the action is Hamiltonian if

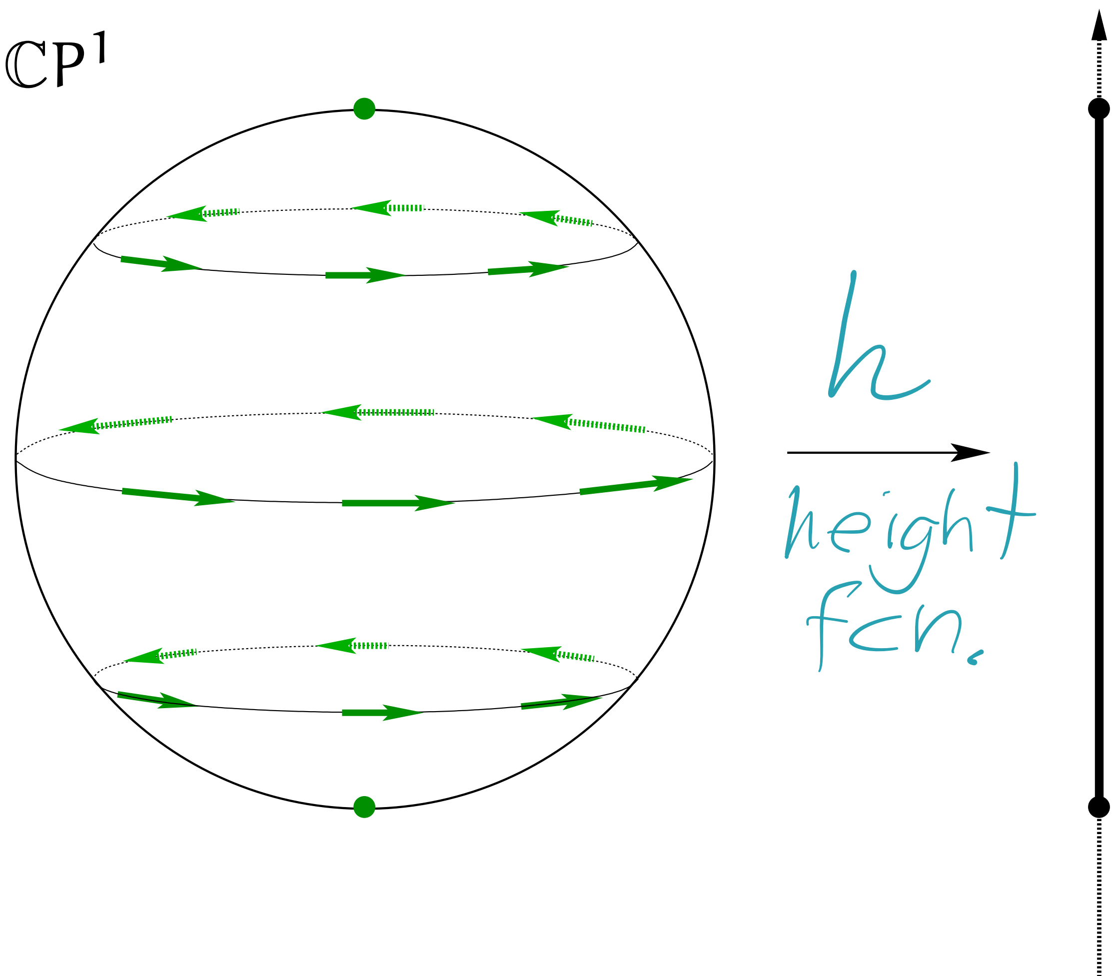
$$\omega(\mathcal{X}_\xi,\cdot) = d\phi^\xi \quad \forall \xi \in \mathfrak{g}$$

Example: $S^1 \circlearrowright M = S^2 = \mathbb{C}P^1$

$$\omega = d\theta \wedge dh$$

$$X_1 = \frac{\partial}{\partial\theta}$$

$$\omega\left(\frac{\partial}{\partial\theta}, \cdot\right) = dh$$



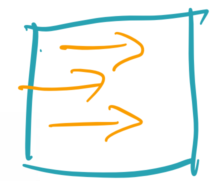
A non-Hamiltonian action

DEF: Suppose $G \circlearrowright (M, \omega)$. We say the action is Hamiltonian if

$$\omega(\mathcal{X}_\xi, \cdot) = d\phi^\xi \quad \forall \xi \in \mathfrak{g}$$

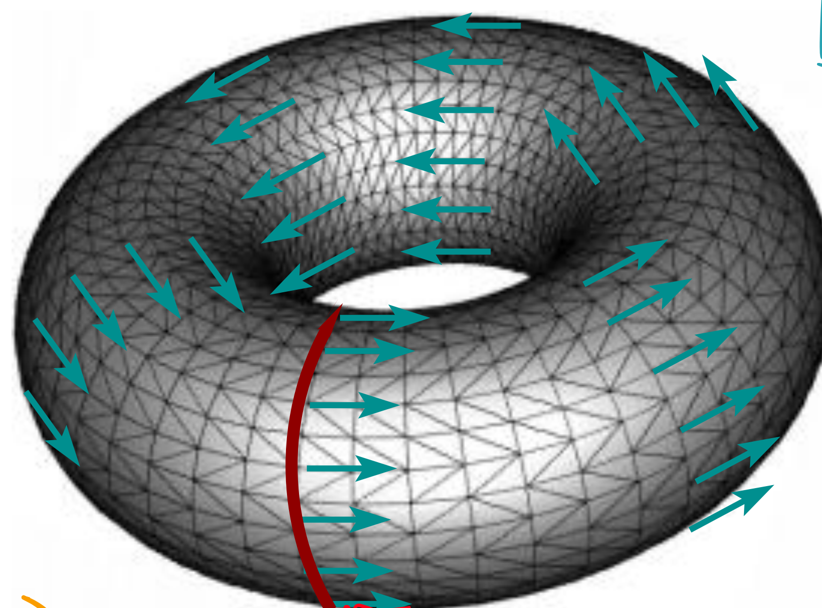
$\mathbb{T}^2 = \mathbb{R}^2 / \mathbb{Z}^2$

Non-Example: $S^1 \circlearrowright T^2 = S^1 \times S^1$ rotating the first factor.



$\omega = dx \wedge dy$

$X_{\mathbb{1}} = \frac{\partial}{\partial x}$



$\omega(X_{\mathbb{1}}, -) = dy$

$\shortparallel$

non-zero in cohomology

P.D to

Hamiltonian actions

DEF: Suppose $G \circlearrowright (M, \omega)$. We say the action is Hamiltonian if

$$\omega(\mathcal{X}_\xi, \cdot) = d\phi^\xi \quad \forall \xi \in \mathfrak{g}$$

$\phi^\xi : M \to \mathbb{R}$ at max/min, $d\phi^\xi = 0$

$\Leftrightarrow$

$X_\xi = 0$

Frankel's Theorem:

A symplectic circle action $S^1 \circlearrowright (M, \omega)$ which preserves the compatible complex structure on a compact Kähler manifold M is

$$\text{Hamiltonian} \iff M^{S^1} \neq \emptyset.$$

crit components are fixed pts

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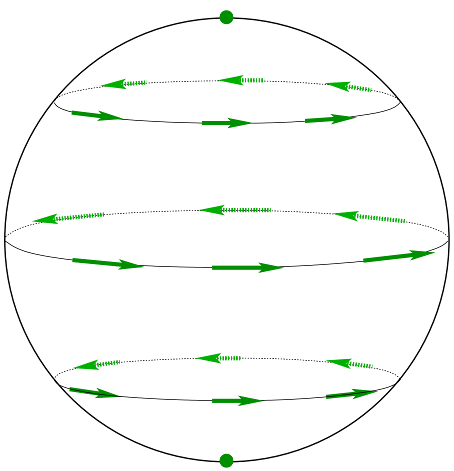

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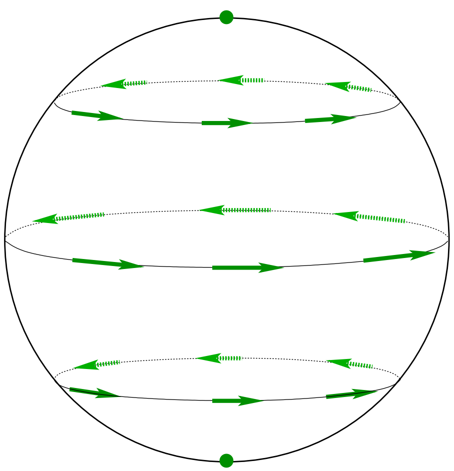

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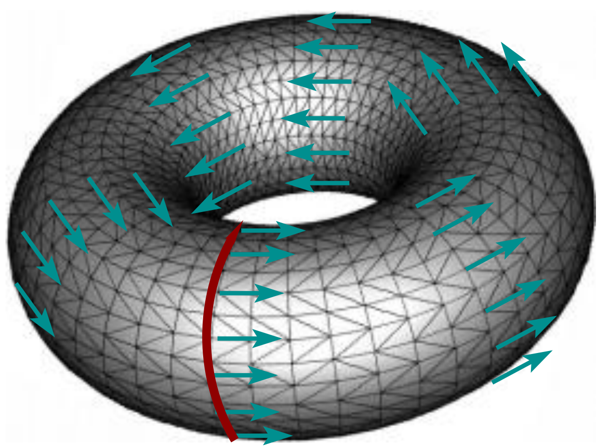

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symplectic + cplx str.

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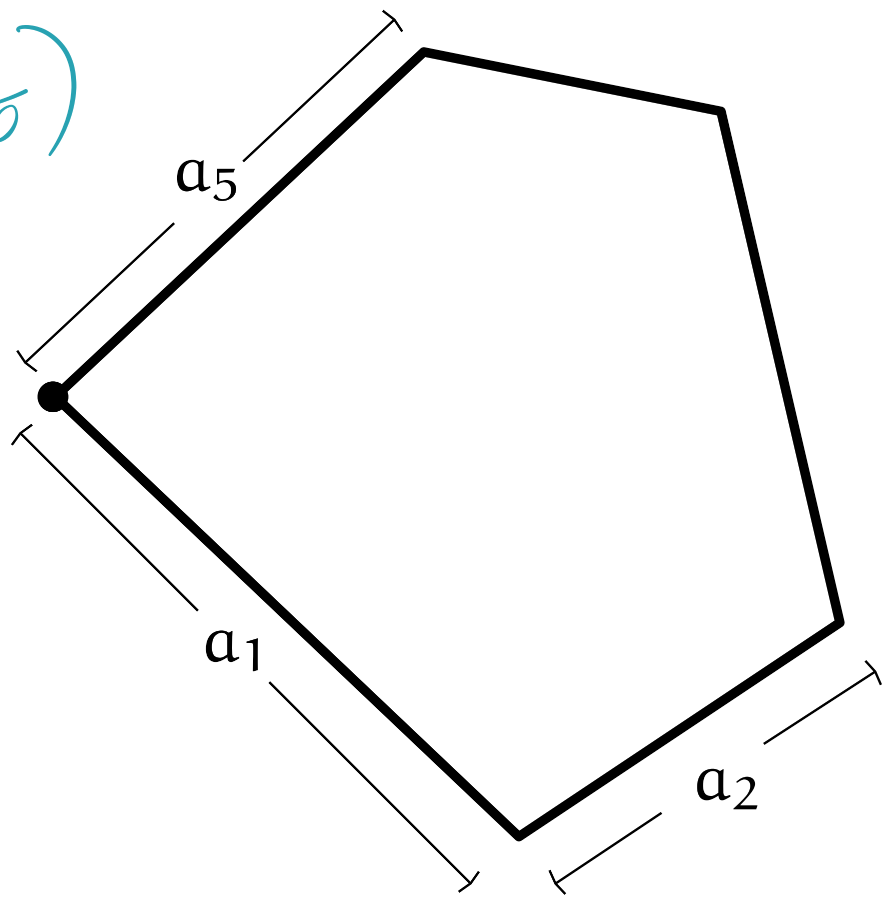
$Pol_3(a_1, \ldots, a_5)$



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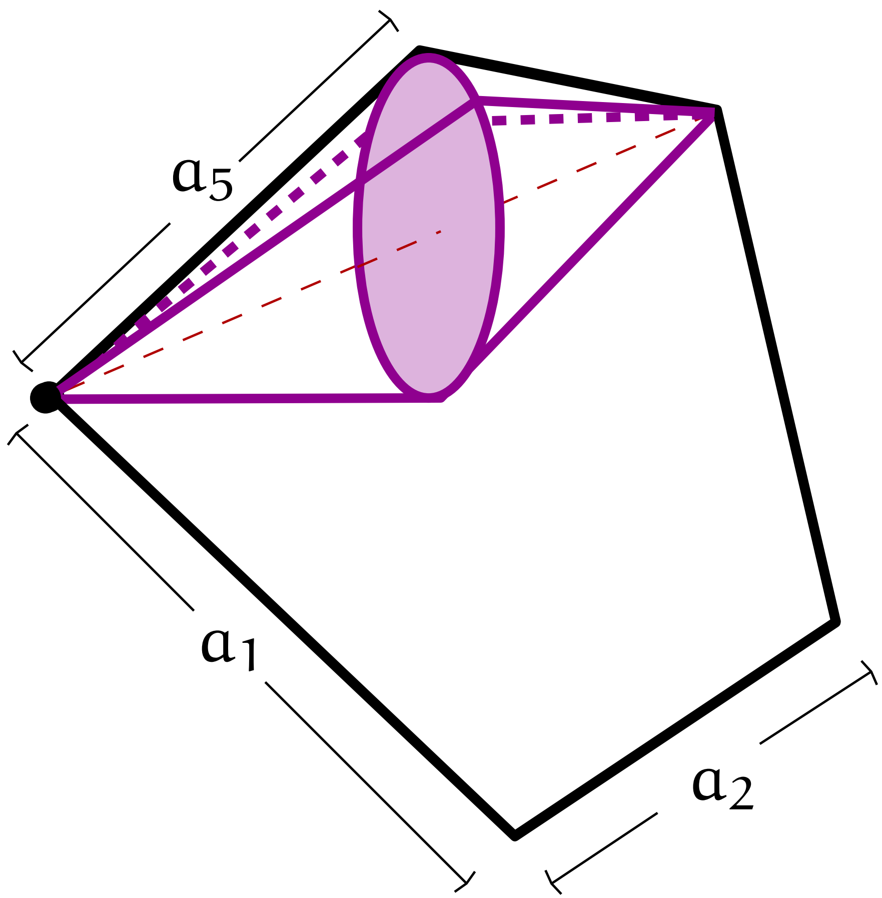

Hamiltonian actions

Frankel's Theorem:

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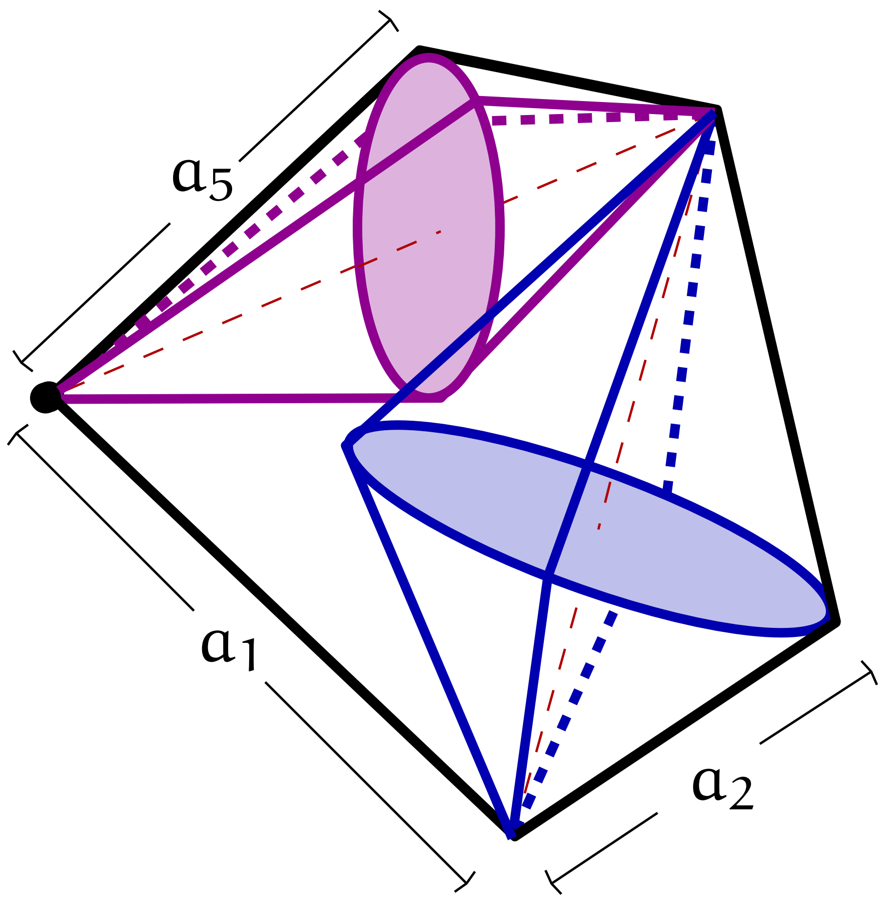

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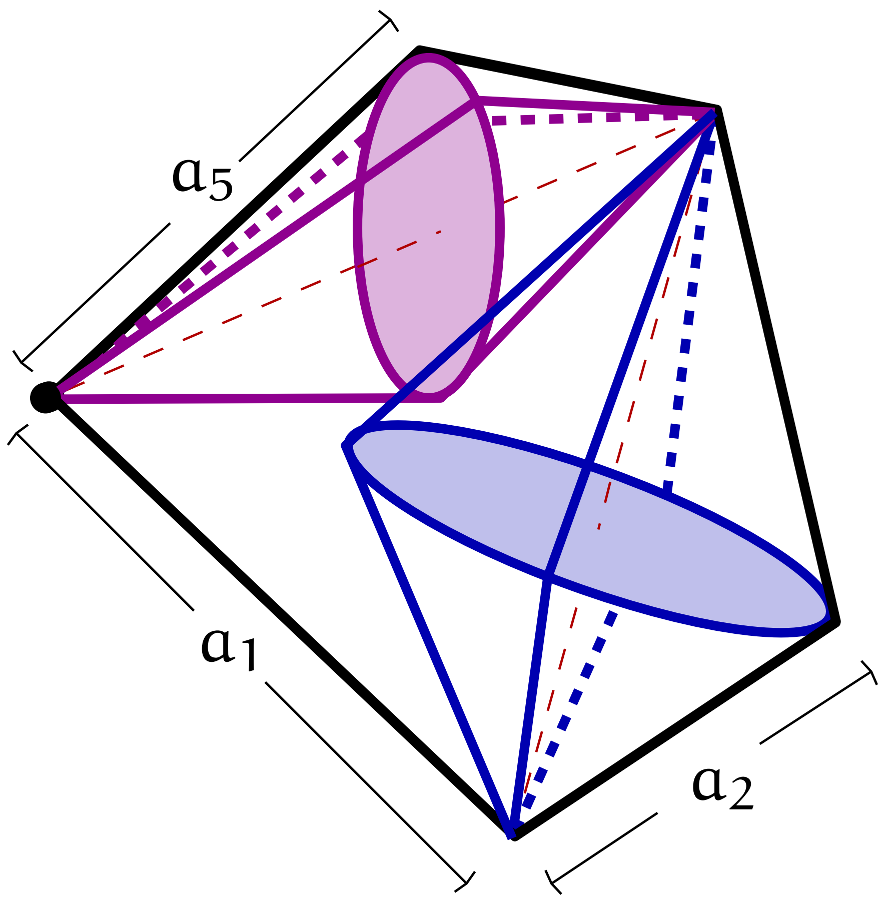

$T^2 \circlearrowright \mathcal{P}ol_3(a_1, \ldots, a_5)$ is Hamiltonian.

Hamiltonian actions

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McDuff's Theorem: M is compact. symplectic

(a) $S^1 \circlearrowright (M^4, \omega) \Longrightarrow$

$$\text{Hamiltonian} \iff M^{S^1} \neq \emptyset.$$

(b) $\exists\, S^1 \circlearrowright (M^6, \omega)$ that has fixed points but is not Hamiltonian.

it has fixed surfaces

- Tolman, Tolman-Jang produced additional examples w/ only isolated fixed points

The moment~~um~~ map

DEF: Suppose $G \circlearrowright (M, \omega)$. We say the action is Hamiltonian if

$$\omega(\mathcal{X}_\xi, \cdot) = d\phi^\xi \quad \forall \xi \in \mathfrak{g}$$

(+ a little more :))

DEF: Combining these for all $\xi \in \mathfrak{g}$, we define the moment~~um~~ map

$$\begin{array}{rcl}\Phi : M & \to & \mathfrak{g}^* \\ p & \mapsto & \left(\begin{array}{ccc}\mathfrak{g} & \longrightarrow & \mathbb{R} \\ \xi & \mapsto & \phi^\xi(p)\end{array}\right).\end{array}$$

$\circlearrowleft G$ coadjoint.

X_ξ

NEED: Φ^ξ can be chosen so that Φ is a G-eq. map.

For tori:

$G = S^1 \times \cdots \times S^1$

ϕ^ξ T-invariant fcn.

The momentum map

DEF: Suppose $G \circlearrowright (M, \omega)$. We say the action is Hamiltonian if

$$\omega(\mathcal{X}_\xi, \cdot) = d\phi^\xi \quad \forall \xi \in \mathfrak{g}$$

DEF: Combining these for all $\xi \in \mathfrak{g}$, we define the momentum map

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<u>Convexity Theorem</u> [Atiyah, Guillemin-Sternberg]:

If $T = (S^1)^d \circlearrowright (M, \omega)$ is Hamiltonian, $\Phi(M)$ is a convex polytope.

$$\underline{\Phi(M) = \mathrm{Conv}(\Phi(M^T)).}$$

COMPACT

FIXED PTS PLAY ROLE!!