Poisson Summer School 2026
University of Antwerp, Belgium
August 3-7, 2026

Symplectic connections

Simone Gutt

(ULB, Brussels)

An affine connection gives a mean to transport a tangent vector -- and more generally any tensor -- from one point to another along a curve in a manifold.

Connections are used in differential geometry, for instance to express differential operators on a manifold in a coordinate free way, or to prove that some subgroups of the diffeomorphism group are Lie groups.

They were first introduced in Riemannian geometry, where one studies a manifold endowed with a metric, i.e. a positive definite symmetric bilinear product in each tangent space depending smoothly on the point. In that situation, there is one and only one affine connection which is torsion free and such that the metric tensor is parallel. It is called the Levi Civita connection and is an essential tool in the study of those geometries.

In symplectic geometry, one studies a manifold endowed with a symplectic structure, which is a non degenerate 2-form, i.e. a skewsymmetric non degenerate bilinear product in each tangent space depending smoothly on the point, asking furthermore this 2-form to be closed. In that case there always exist affine connections which are torsion free and such that the symplectic 2-form is parallel, but there is no unicity.

This course on symplectic connections has three parts:

Part 1 contains the definition of a symplectic connection on a symplectic manifold, gives properties of the curvature of such a connection, describes possible choices of symplectic connections with examples, and includes properties of the space of all symplectic connections.

In Part 2 we briefly recall the theory of deformation quantization and show how symplectic connections are linked to deformation quantization in the symplectic framework.

Part 3 studies particular classes of symplectic connections. We show that imposing conditions on the curvature of a symplectic connection, such as being of Ricci-type, gives very strong rigidity properties.