

Integrable birational discretisations from a 3D generalisation of QRT

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When completely integrable Hamiltonian systems are discretised, the resulting discrete time systems are often no longer integrable themselves. This is the so-called *problem of integrable discretisation*. Two known exceptions to this situation in 3D are the Kahan discretisations of the Euler top and the Zhukovski-Volterra gyrostat with one non-zero linear parameter β , both birational maps of degree 3. The integrals of these systems define pencils of quadrics. By analysing the geometry of these pencils, we develop a framework that generalises QRT maps and QRT roots to 3D, which allows us to create new integrable maps as a composition of two involutions. We show that under certain geometric conditions, the new maps become of degree 3. We use these results to create new families of discrete integrable maps and we solve the problem of integrability of the Zhukovski-Volterra gyrostat with two β 's.

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