What does the Alexander polynomial know about minimal surfaces?

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In 1928 Alexander defined a polynomial invariant of knots and links. It is combinatorial in nature and can be determined directly from a plane diagram of the link. I will explain a conjecture which says that the coefficients of the Alexander polynomial is in fact a count of minimal surfaces: we think of a link L as lying in the 3-sphere at infinity of hyperbolic 4-space and then count connected complete minimal surfaces in hyperbolic 4-space which have ideal boundary equal to L. If true, this would mean that the relatively simple combinatorial computation of the Alexander polynomial would imply the existence of minimal surfaces, so give solutions to a non-linear PDE! I will outline a strategy for the proof of this conjecture which goes via 6-dimensional symplectic geometry and J-holomorphic curves.