Bifurcation diagrams of integrable systems, smooth and with kinks

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The integrals of a Liouville-integrable system confine every orbit to a joint level set, and the bifurcation diagram, the set of critical values of the momentum map F, records where these level sets change. A rank drop of dF is a critical point of c·F for some conormal c. With c as an unknown, the rank-0 points, the rank-1 ribs and, for three degrees of freedom, the rank-2 sheets become solution sets of polynomial Lagrange systems. These can be continued numerically by pseudo-arclength.

We certify the list of rank-0 points with Gröbner bases and their Williamson types with Sturm sequences, and we locate parabolic and Hamiltonian–Hopf points along ribs. We then consider momentum maps that are only piecewise smooth, F = A + B|e|: the Hamiltonian is continuous and the force jumps across a wall. Energy is still conserved, the flow is still symplectic, and a wall crossing acts as a symplectic transvection. On the wall the relevant system is the Lagrange system of F|_Σ, from which B drops out; a bound on its multiplier splits it into corner points (Clarke-critical, Karush–Kuhn–Tucker points), rib endpoints and grazing points. For one degree of freedom this is Ekeland's classification of 1977. For n ≥ 2 we have not found these wall strata in the literature. We compute them for several examples: a V-shaped champagne bottle, a Jaynes–Cummings system with a kink, a magnetic step, and a system with three degrees of freedom.