

Miniworkshop
“Geometry & Mechanics:
Differential systems and related topics”
University of Antwerp, Belgium
Sept 17-18, 2026

Exterior differential systems on Lie algebroids
and their applications

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In recent years, there has been increased interest in a coordinate independent geometric theory for studying solutions of systems of PDEs, known as the theory of exterior differential systems (EDS). This theory originates from the ideas of several mathematicians, such as Pfaff, Darboux, Goursat and, most prominently, Cartan. Here, the PDEs are represented by a collection of differential forms on a manifold, satisfying the condition that the collection is closed under exterior differentiation. The main motivation to study EDS is to compute integral manifolds, which are submanifolds satisfying the property that all differential forms in the collection vanish. Since any system of partial differential equations can be written as an EDS (with an independence condition), finding solutions to a system of PDEs can be given the desired geometric interpretation, namely, finding integral manifolds of its associated EDS.

In this thesis, we extend the theory of EDS from manifolds (and their tangent bundles), to Lie algebroids. To be more precise, we extend the theory so that instead of limiting ourselves to finding submanifolds such that a collection of differential forms on a manifold vanishes (i.e., a form on the tangent bundle), we will find submanifolds so that certain forms on a vector bundle vanish. However, to make sense of the fact that the collection of forms should be differentially closed, we need to assume that this vector bundle has some additional structure, namely, that of a so-called Lie algebroid, where an exterior derivative operator is defined.

First, we focus on existence results for integral manifolds, in the case where the Lie algebroid is transitive. In particular, we generalize two versions of the celebrated Cartan–Kähler theorem from EDS on manifolds to this setting. We then motivate our extension by giving many applications. We find applications in b-symplectic geometry (using an EDS on a b-tangent bundle), and in group-invariant solutions to dynamical systems with symmetry (using an EDS on an Atiyah Lie algebroid). We conclude with a detailed discussion of the invariant inverse problem, where we demonstrate that it can be reformulated as an EDS problem on a very specific Lie algebroid, called the IP Lie algebroid.