Lifting Hamiltonian torus actions to integrable systems

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An *integrable system* consists of a $2n$-dimensional symplectic manifold (M, ω) equipped with a set of n real-valued functions which are independent and Poisson commute. In the case that the Hamiltonian flow of these functions generates an effective action of the n-torus, the system is called *toric*. Toric integrable systems are well-understood thanks to results of Atiyah, Guillemin-Sternberg, and Delzant which classify them in terms of a certain class of polytopes.

Similarly, a *complexity-one space* is the Hamiltonian action of an $(n-1)$-torus, so, roughly, it can be thought of as a set of $n-1$ independent, Poisson commuting functions. In particular, a complexity-one space on a 4-manifold is a Hamiltonian S^1-action. This talk is concerned with various versions of the following question: given a complexity-one space generated by functions $g_1, \ldots, g_{n-1}$, when can one additional function f be found so that $(g_1, \ldots, g_{n-1}, f)$ forms an integrable system of a certain desirable type?

We will start with the case that $n = 2$ by reviewing a result by Karshon about which S^1-actions can be extended to a toric system on a 4-manifold, and then explaining a recent result, that I obtained jointly with S. Hohloch, about a class of integrable systems to which all S^1-actions can be lifted. I will also explain recent results, joint with S. Tolman and Y. Liu, about lifting complexity-one spaces to toric systems in higher dimensions, generalizing Karshon's result.