Miniworkshop "Clifford analysis and symplectic dynamics"
University of Antwerp, Belgium
September 4-5, 2023

Positive energy representations of gauge groups

Bas Janssens

(TU Delft)

Elementary particles can be described by projective unitary representations of the relevant group of symmetries. In the absence of internal degrees of freedom, this group of symmetries is the Poincaré group SO(d-1,1) $\rtimes$ R^d, and a classical result of Wigner states that every irreducible representation is labelled by an SO(d-1,1)-orbit in R^d, together with an irreducible representation of the `little group', the stabiliser group of a base point in this orbit. The physically relevant representations are those that are of `positive energy', in the sense that every timelike translation is implemented by a generator with positive spectrum.

In the presence of internal degrees of freedom, the group of symmetries is substantially larger: it is the semidirect product of the Poincaré group with the `gauge group', the group of vertical automorphisms of a principal fibre bundle. Although the classification of all projective unitary representations of this infinite-dimensional Lie group is currently wide open, it turns out that the extra assumption of `positive energy' adds just enough analytic control to approach this problem. We show that the representations then localise at the boundary of the conformal compactification of Minkowski space R^d, leading to a complete classification in dimension d>2. (Joint work with Karl-Hermann Neeb)