Functional inequalities on Lie groups

(Based on Joint works with A. Kassymov and M. Ruzhansky)

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The presentation will be about a family of logarithmic inequalities on several of Lie groups. The inequalities we will discuss are well-known in the Euclidean space R^n and we will discuss their extension in the general setting of stratified groups and sometimes in the even more general setting of graded or even general Lie groups; recall that:

$$\text{general Lie groups} \supset \text{homogeneous} \supset \text{graded} \supset \text{stratified} \supset \{R^n, \text{Heisenberg}\}.$$

These inequalities as in [1], [2] and [3] include: the log- Sobolev, Shannon, log- Gagliardo-Nirenberg, and Nash inequalities. Interestingly, restricting ourselves to the stratified setting, we also show the "semi-Gaussian" inequality which recovers the classical Gross inequality when the stratified group is simply R^n. In the case of the Heisenberg group, the appearing constant allows us to pass to infinite dimensions, which in turn might give rise to an infinite-dimensional Heisenberg group with the first stratum of infinite dimension and a probability measure on it.

[1] M. Chatzakou and A. Kassymov and M. Ruzhansky. Anisotropic Shannon inequality. To appear in: Osaka J. Math.

[2] M. Chatzakou and A. Kassymov and M. Ruzhansky. Logarithmic Sobolev inequalities on Lie groups. arXiv:2106.15652.

[3] M. Chatzakou and A. Kassymov and M. Ruzhansky. Logarithmic Sobolev & Poincaré inequalities on the Heisenberg group. preprint.

[4] M. Chatzakou, S. Federico and B. Zegarlinski, q-Poincaré inequalities on Carnot groups with filiform type Lie algebra. To appear in: Potential Anal.