

# Verification Conquers Fault Tree Analysis

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# Reliability

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# Reliability Engineering

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- ▶ Risk analysis ensures that critical assets, like medical devices and nuclear power plants, operate in a safe and reliable way.
- ▶ Fault tree analysis (FTA) is one of the most prominent techniques.
- ▶ Used by a wide range of industries (aerospace, automotive, nuclear, medical, process engineering)
- ▶ Used by many companies and institutions: FAA, NASA, ESA, Airbus, Honeywell, etc.
- ▶ Industrial standards by the IEC and by ISO for automotive applications

# The SpaceX Falcon-9 Explosion

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**Elon Musk**  @elonmusk · 28 Jun 2015

There was an overpressure event in the upper stage liquid oxygen tank. Data suggests counterintuitive cause.

 471    4.1K    3.3K



**Elon Musk** 

@elonmusk

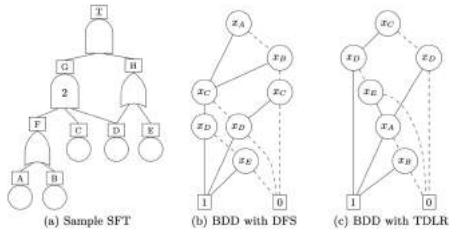
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That's all we can say with confidence right now. Will have more to say following a thorough fault tree analysis.

A launch failure in 2015 resulted in a loss of a quarter billion dollars

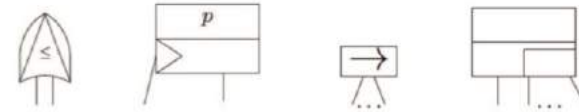
# Talk Overview

1.



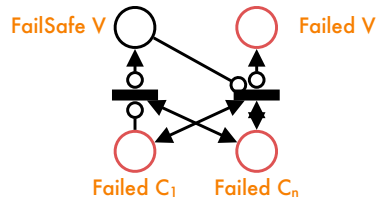
## Classical Static Fault Trees

2.



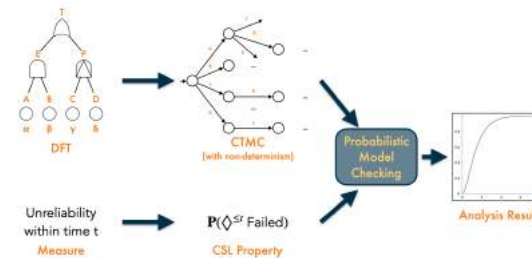
## Dynamic Fault Trees

3.



## Petri Net Semantics

4.



## Scaling Up DFT Analysis

5.



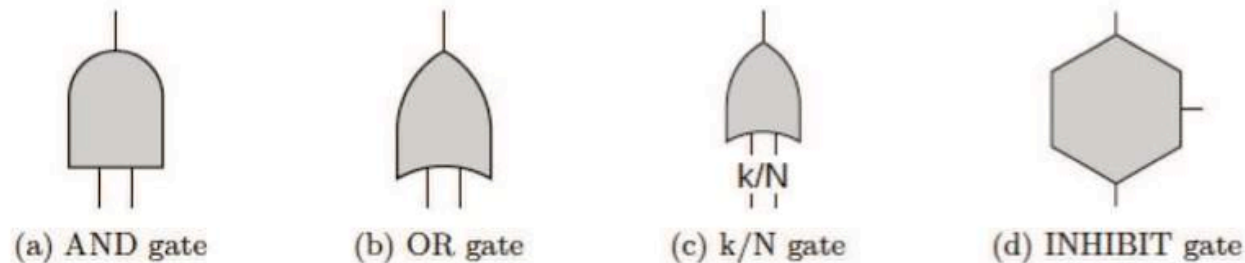
## Industrial Case Studies

6.



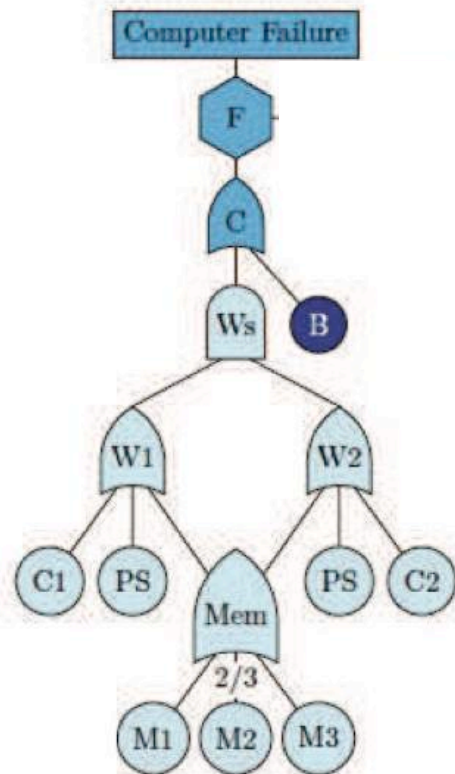
## Commercialisation

- ▶ Fault tree is a **directed acyclic graph** consisting of two types of nodes: **events** (depicted as circles) and **gates**:

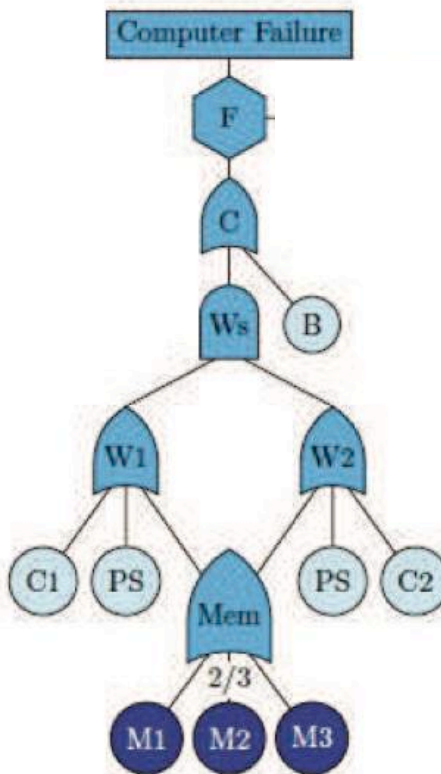


- ▶ An **event** is an occurrence within the system, typically the failure of a component or sub-system.
- ▶ Events can be divided into:
  - ▶ **basic** events (BEs), which occur on their own, and
  - ▶ **intermediate events**, which are caused by other events
- ▶ The root, called the **top level event** (TLE), models a system failure

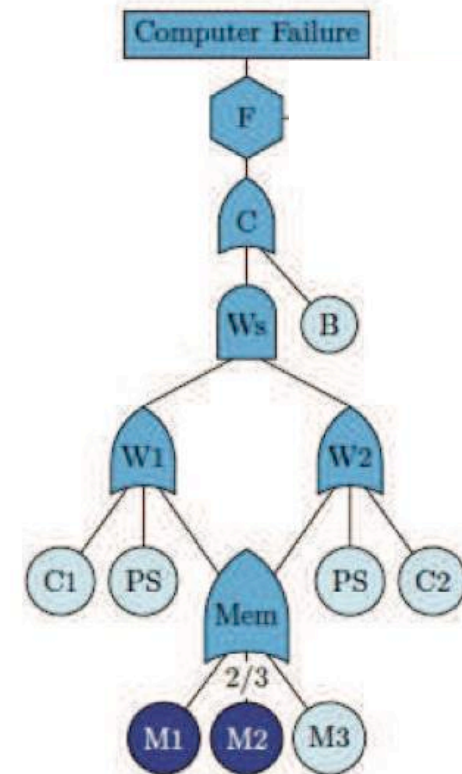
# Minimal Cut Sets



minimal



not minimal

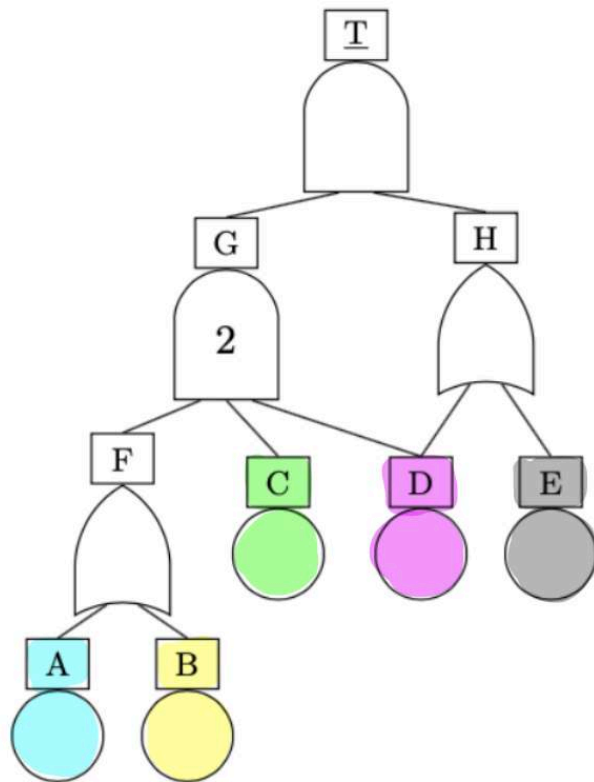


minimal

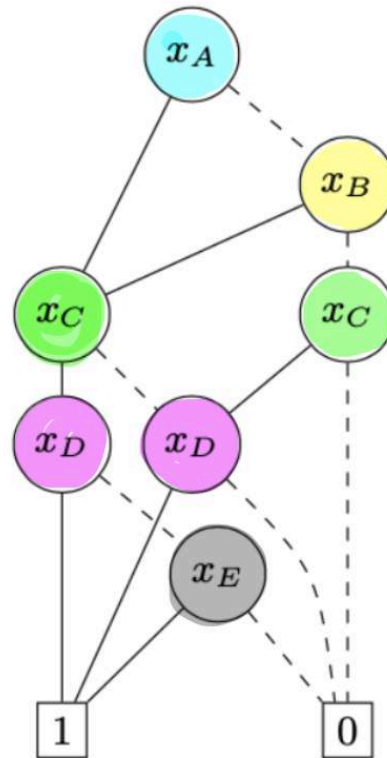
A **cut set** is a set of components that together can cause the system to fail.

A **minimal** cut set is a cut set without proper subset being a cut set.

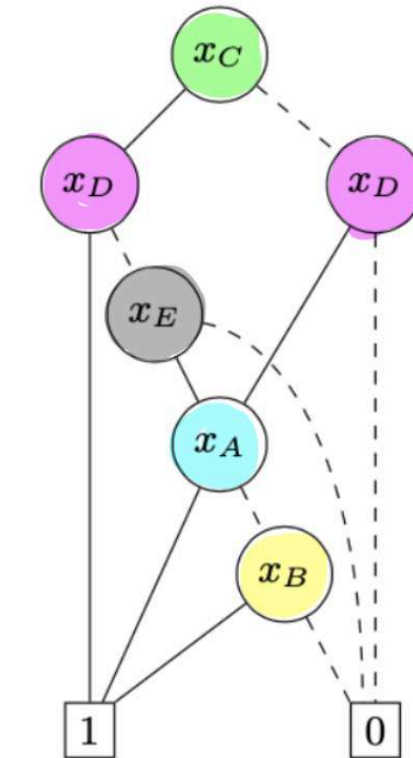
# SFT Analysis



(a) Sample SFT



(b) BDD with DFS



(c) BDD with TDLR

- Turn SFT into **propositional logic formula**
- Encode as a **binary decision diagram**
- Calculate **minimal cut sets, MTTF, reliability and sensitivity** using BDDs



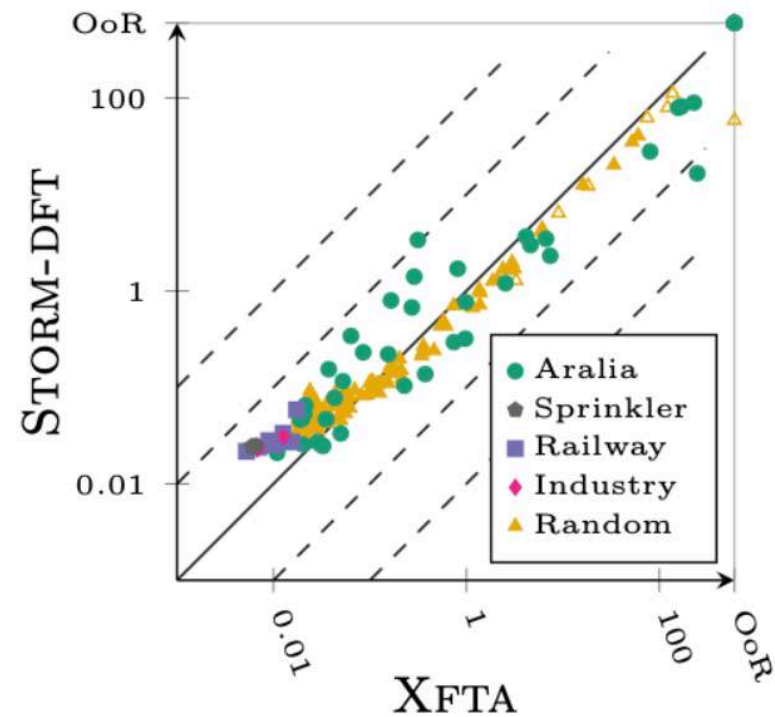
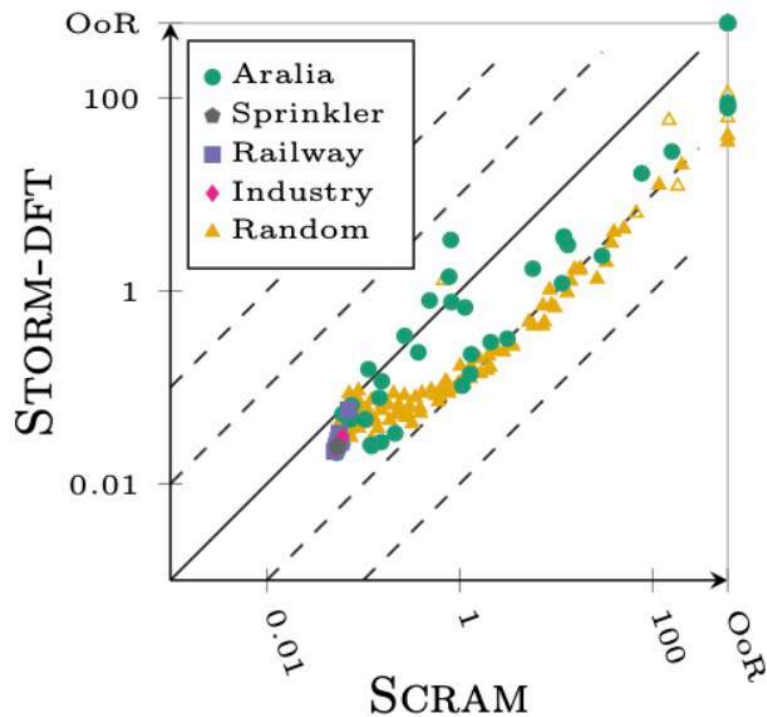
# Experiments: Computing MCS

[Basgöze et al., NASA FM 2022]

	Aralia	Sprinkler	Railway	Industry	Random	Random (Large)
#BEs	25-1567	31	22-54	36-184	150	500
#Gates	20-1622	35	69-259	21-67	70-122	261-316



all run times in seconds



Storm-DFT computes MCSs faster than XFTA and SCRAM for large SFTs

# SFT Deficiencies

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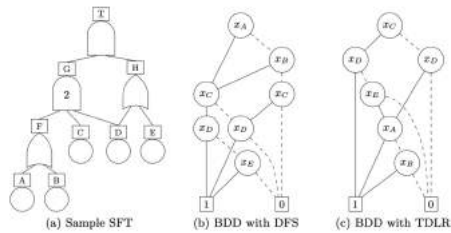
- **Their simplicity**
  - simple to comprehend and analyse
  - too simple to model realistic scenarios
- **Lack of common dependability patterns**
  - spare management
  - functional dependencies (e.g., common-cause failures)
  - redundancies
- **Static behaviour**
  - no temporal orderings of faults
  - top-level event only depends on set of failed events

## Many variants:

state-event fault trees, boolean-logic driven Markov processes,  
SD fault trees, PANDORA fault trees, Dugan's dynamic fault trees

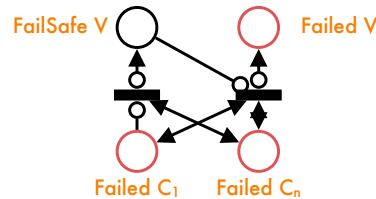
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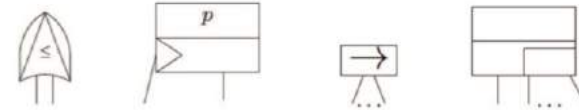
## Petri Net Semantics

5.



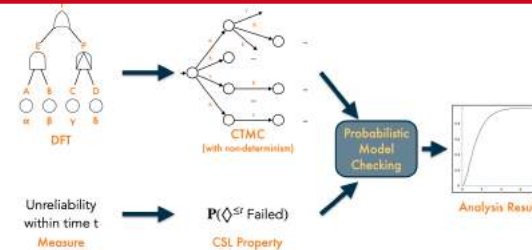
## Industrial Case Studies

2.



## Dynamic Fault Trees

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## Scaling Up DFT Analysis

6.



## Commercialisation

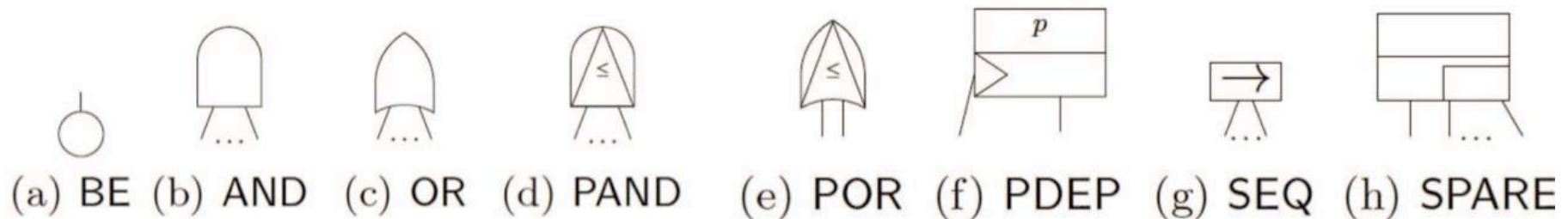
# Dugan's Dynamic Fault Trees

2000 IEEE Reliability Society Award



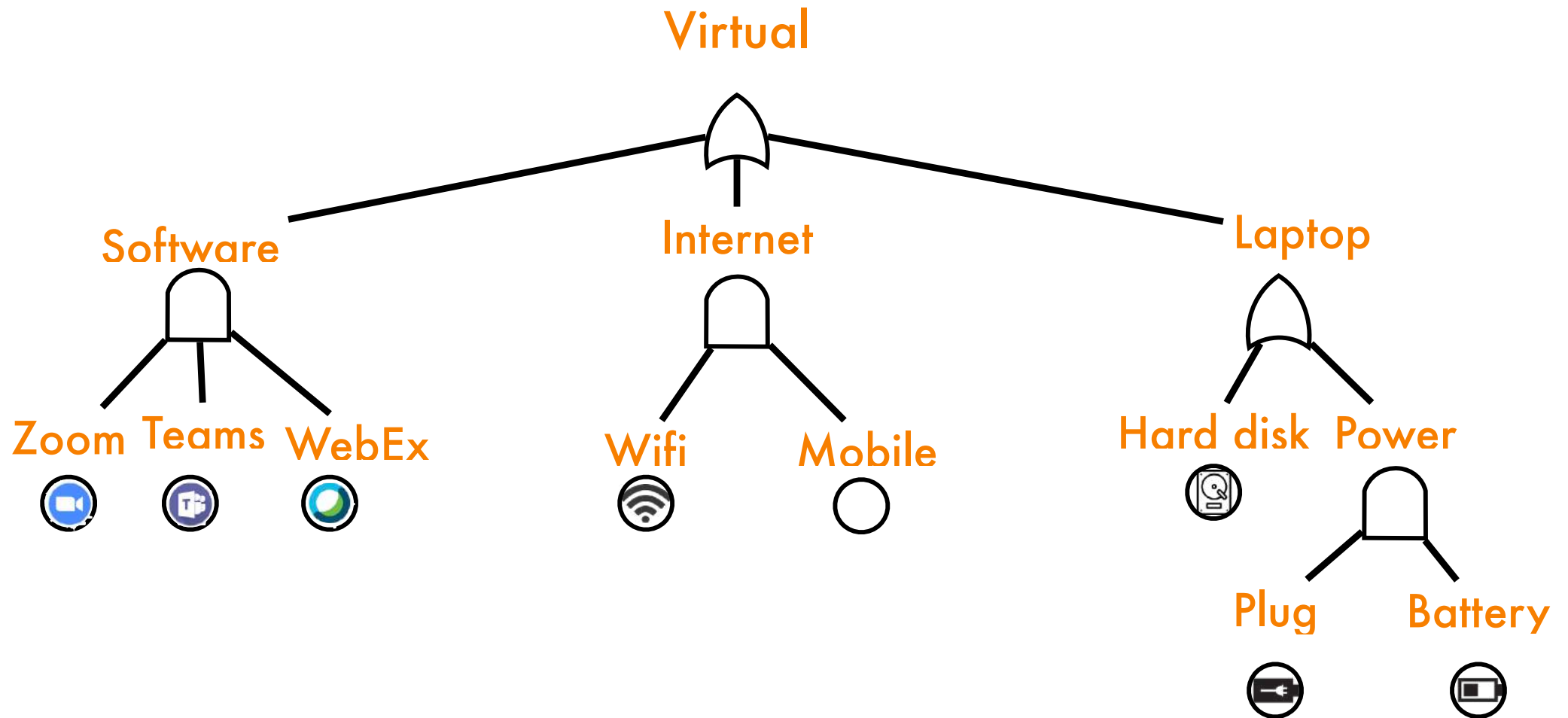
*"Dynamic fault tree analysis has extended the state of the art and the state of the practice in analysis of the dependability of computer systems."*

- JOANNE BECHTA DUGAN, PROFESSOR OF ELECTRICAL & COMPUTER ENGINEERING

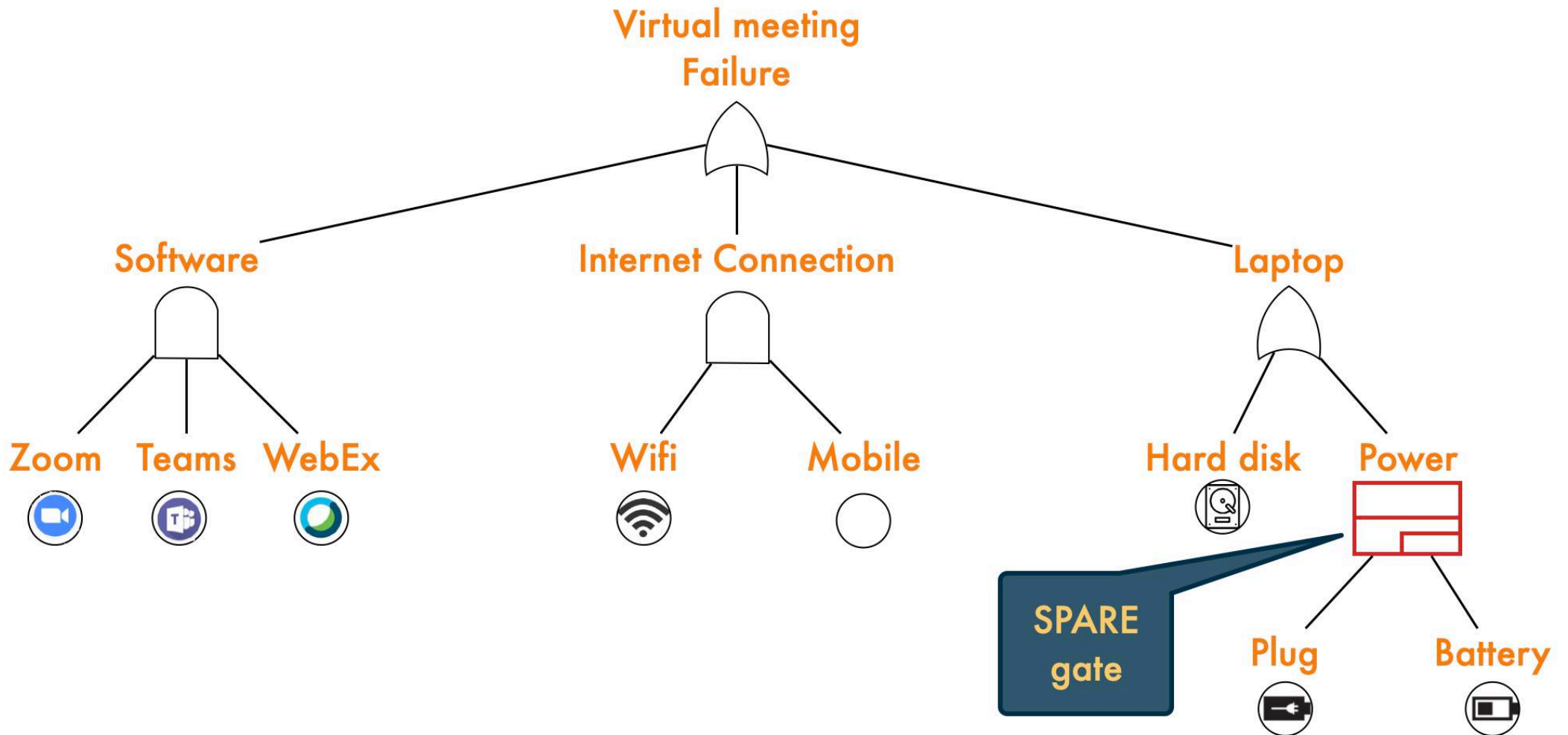


**Galileo User's Manual & Design Overview**

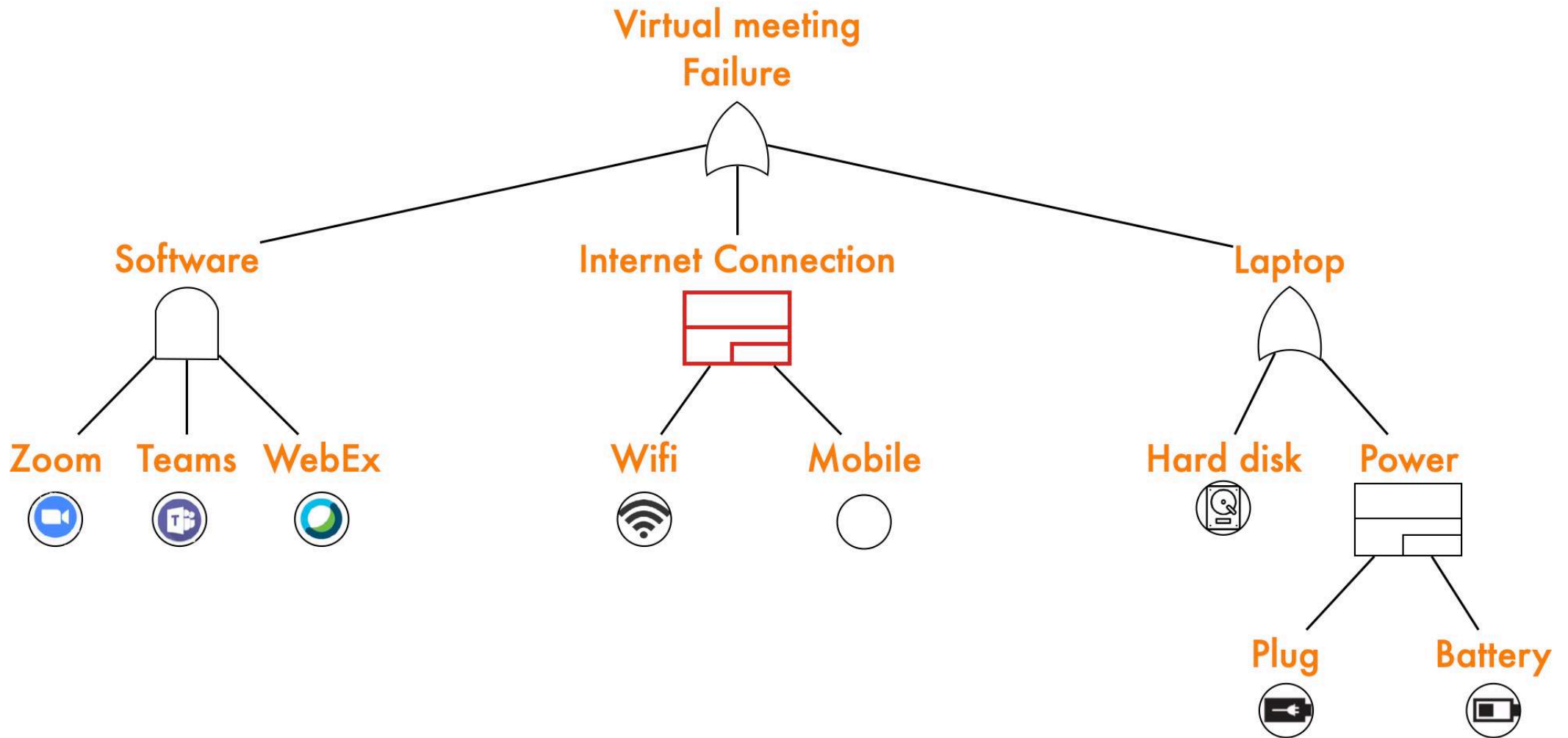
# A Sample Dynamic Fault Tree



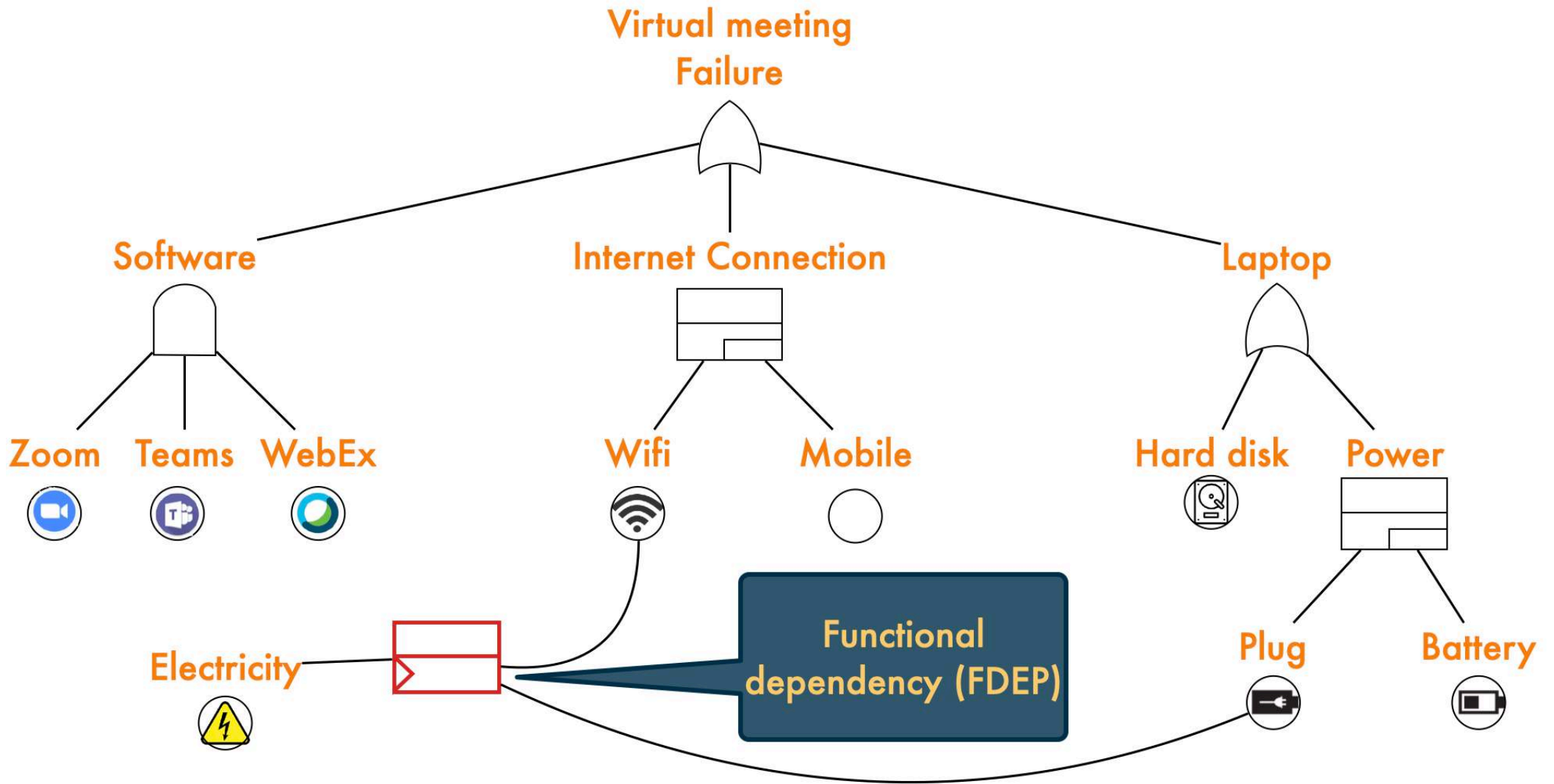
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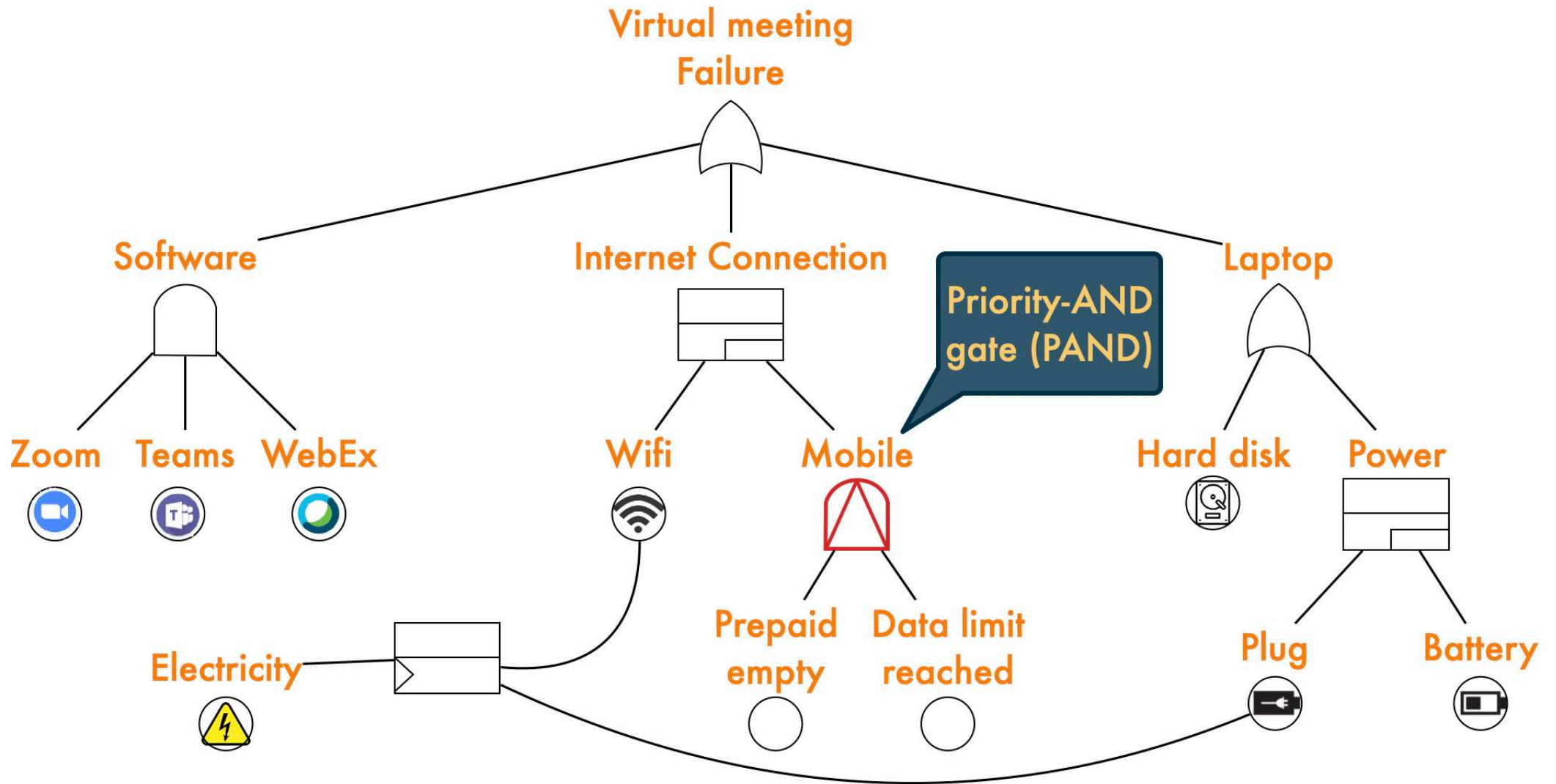


# A Sample Dynamic Fault Tree





# A Sample Dynamic Fault Tree



## No Free Lunch

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- Minimal cut set analysis not applicable
  - generalisation to cut sequences insufficient
  - the behaviour of a DFT is history-dependent
- Analysis by generating stochastic (decision) process
  - continuous-time Markov chains/decision processes
  - other approaches: via Bayesian networks, Petri nets
- Use Markov chain analysis to obtain measures

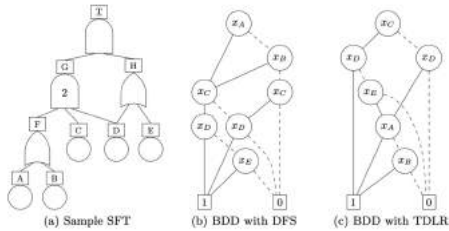


*„The construction of a Markov model for any but the simplest system is tedious and error prone.“*

[Dugan et al., 1992]

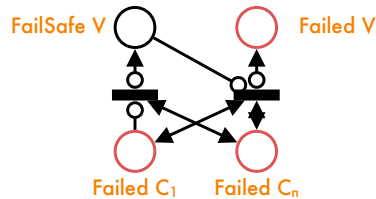
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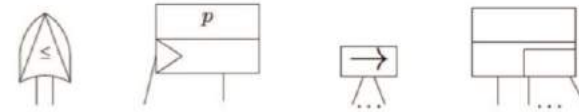
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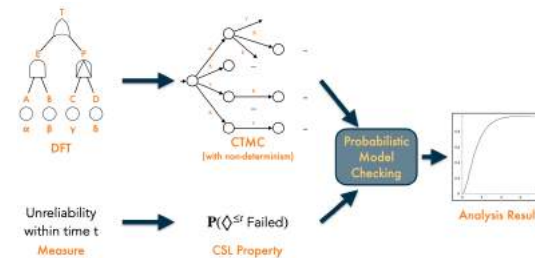
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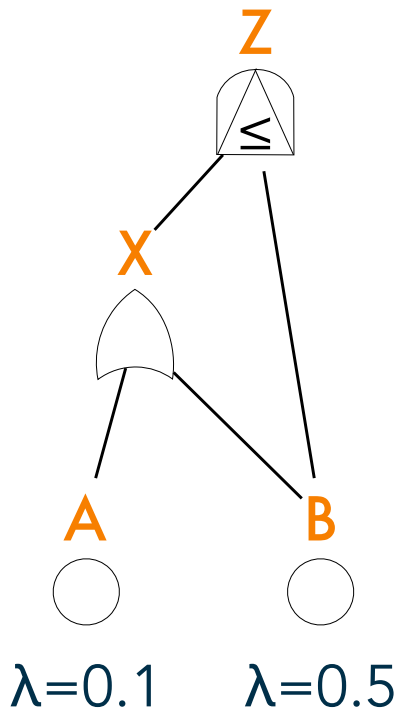


## Scaling Up DFT Analysis

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## Commercialisation



✓ Unreliability within 1 time unit

✓ DFTCalc **0.0175**

✓ Storm **0.3935**

✓ Different semantics for failure propagation

✓ Semantic issues when combining gates

Expressing gates with other gates

Simultaneous failures in priority gates

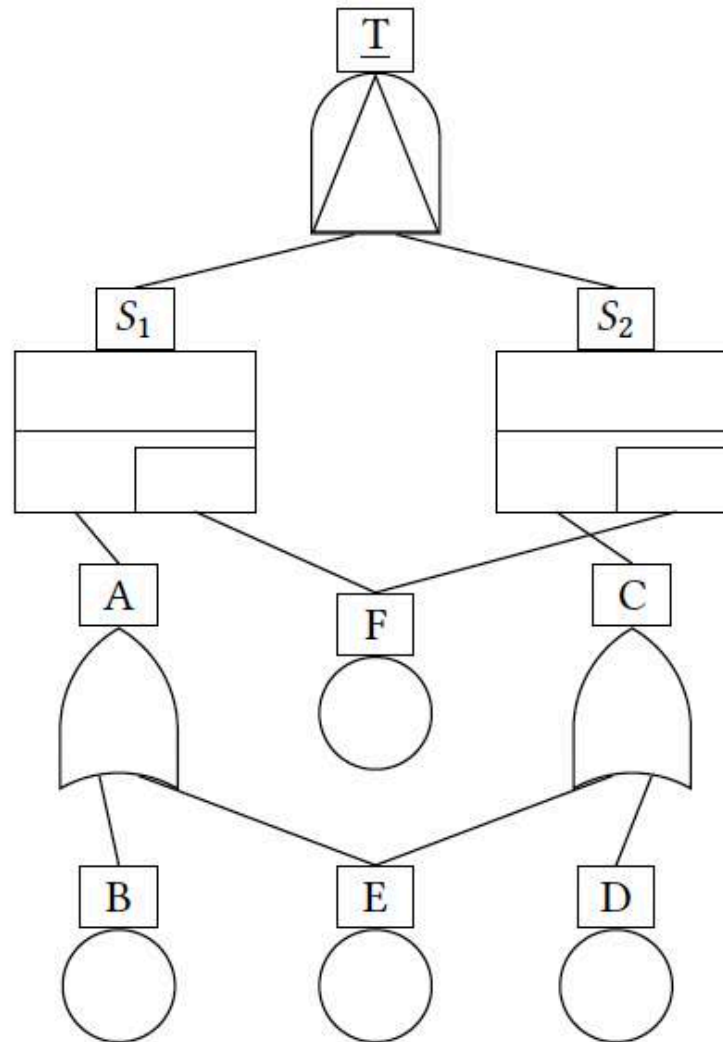
SPARE races

Nested SPARE gates

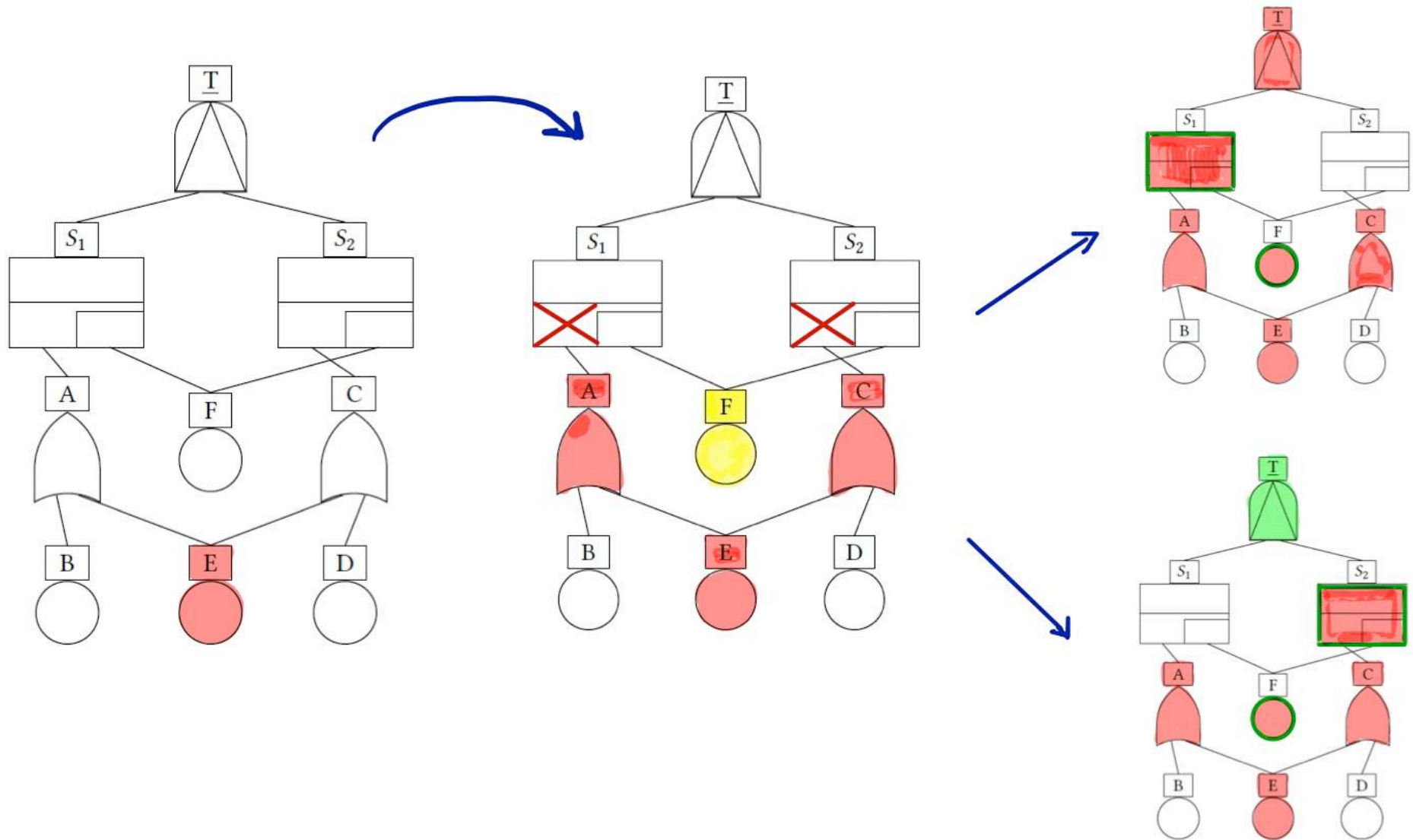
Combining SEQ and FDEP

# Spare Races

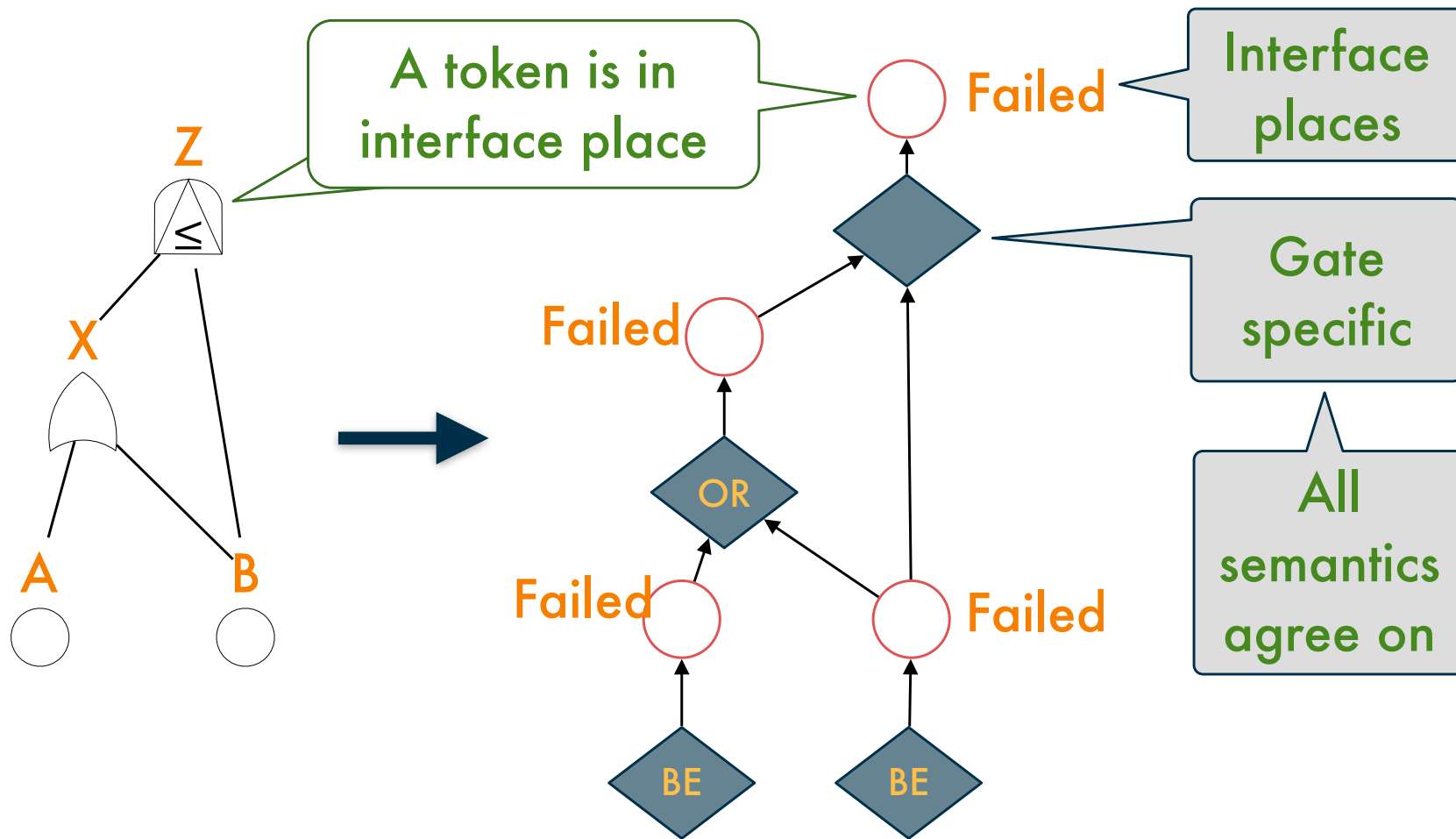
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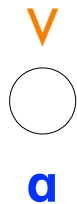


# A Petri Net Approach

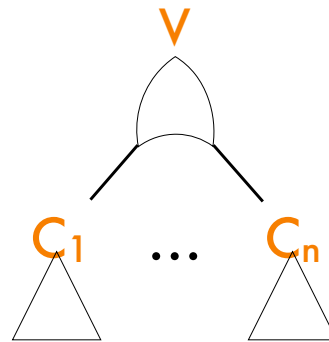


# Petri Net Templates

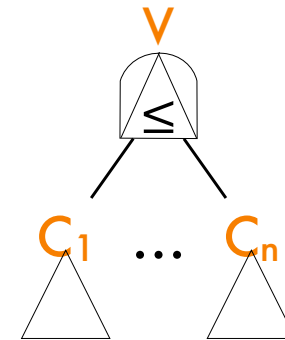
## Basic Event



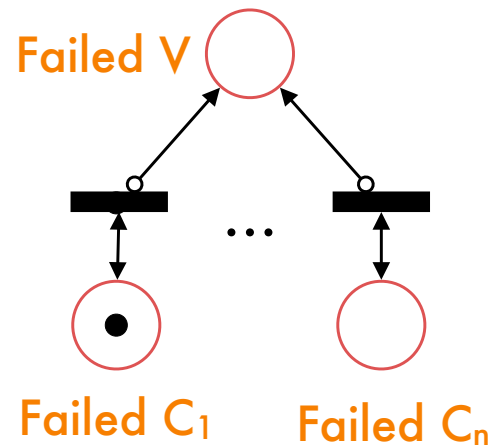
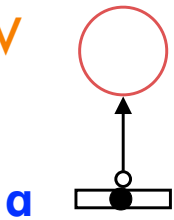
## OR-gate



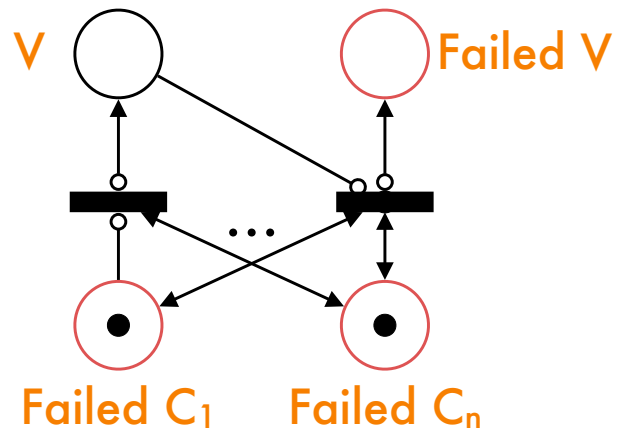
## Priority-AND



## Failed $V$



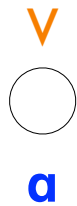
## FailSafe $V$



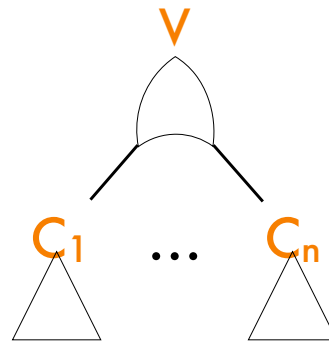


# Petri Net Templates

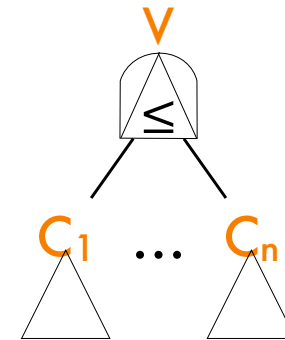
## Basic Event



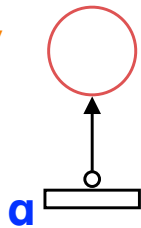
## OR-gate



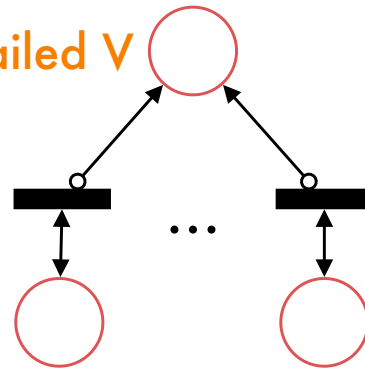
## Priority-AND



## Failed V



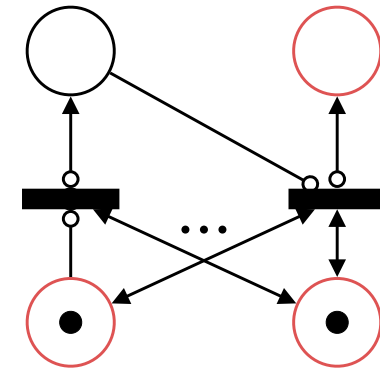
## Failed V



## Failed $C_1$

## Failed $C_n$

## FailSafe

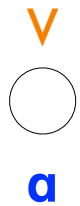


## Failed $C_1$

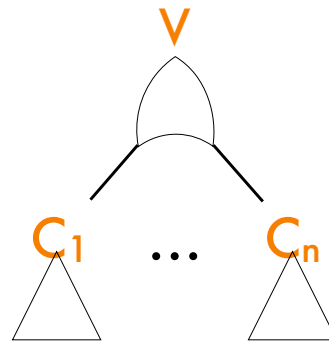
## Failed $C_n$

# Petri Net Templates

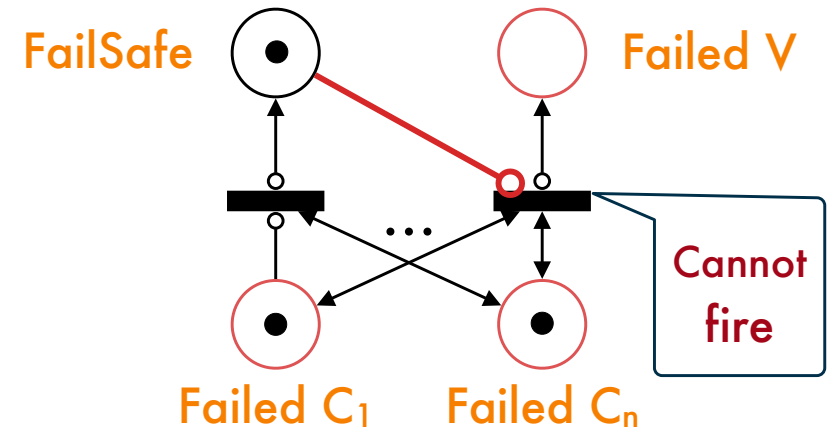
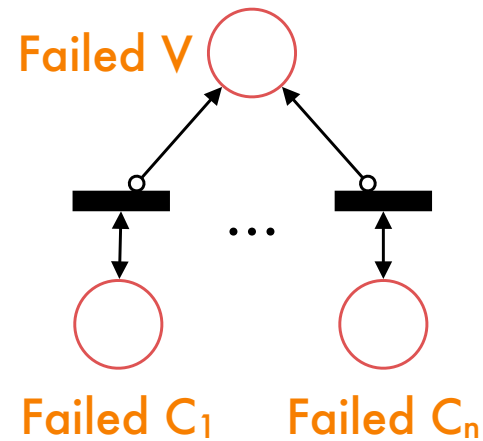
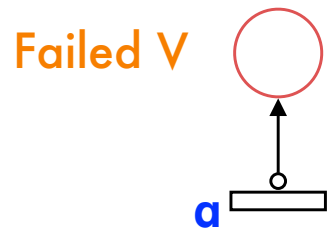
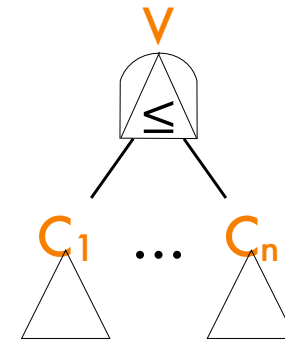
## Basic Event



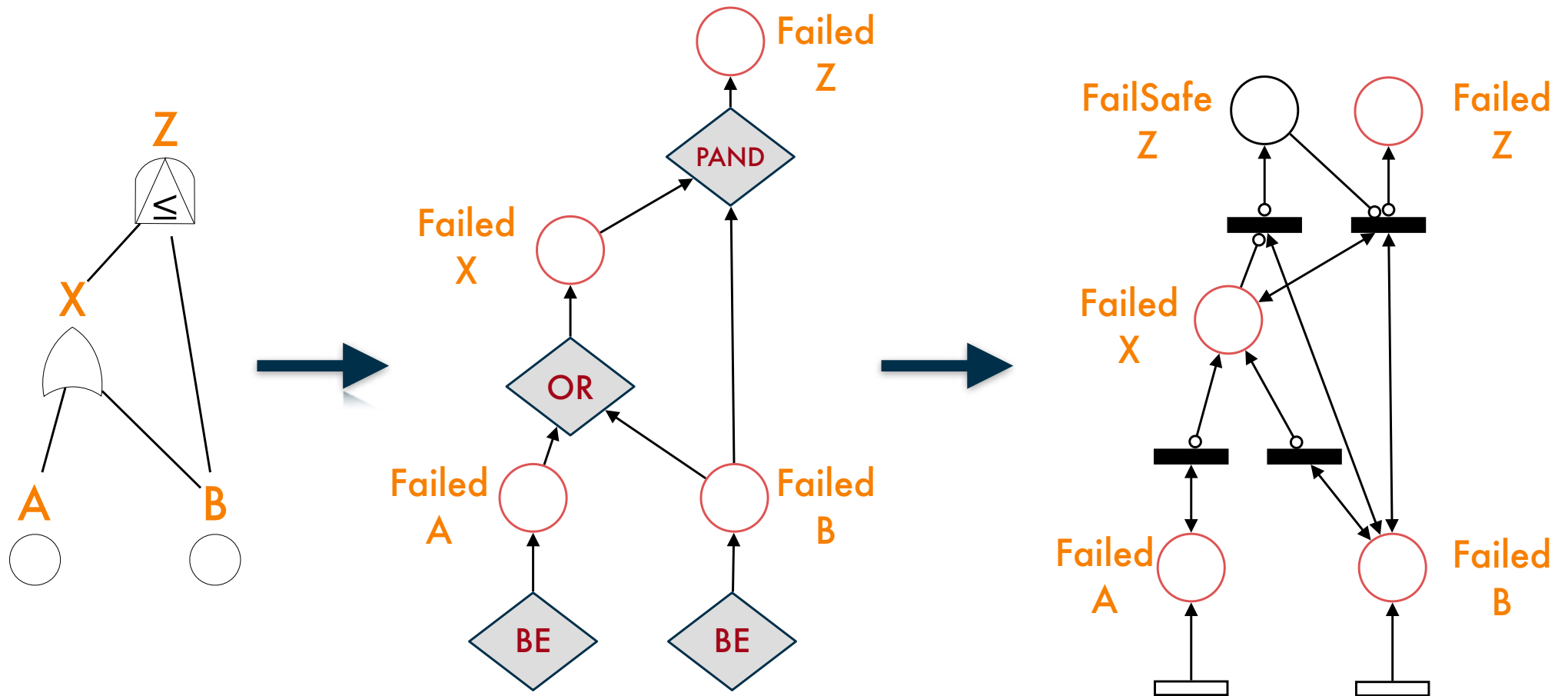
## OR-gate



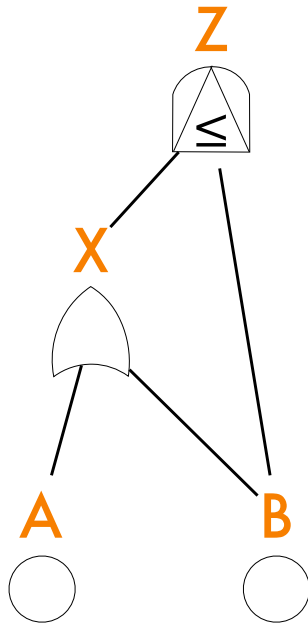
## Priority-AND



# A Petri Net Approach

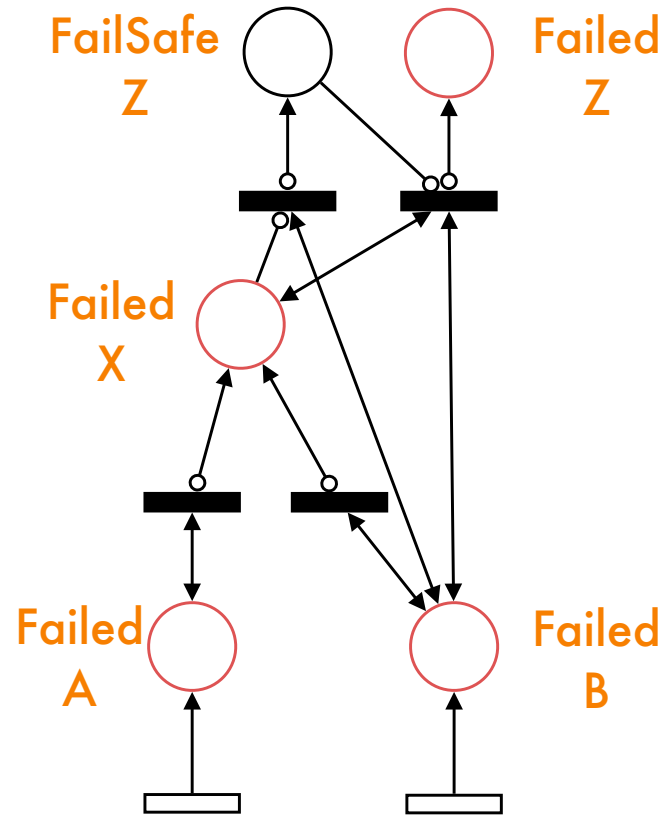


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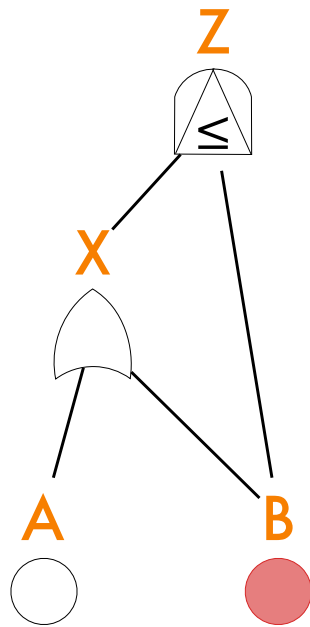


$\tau_1 =$

$\tau_2 =$

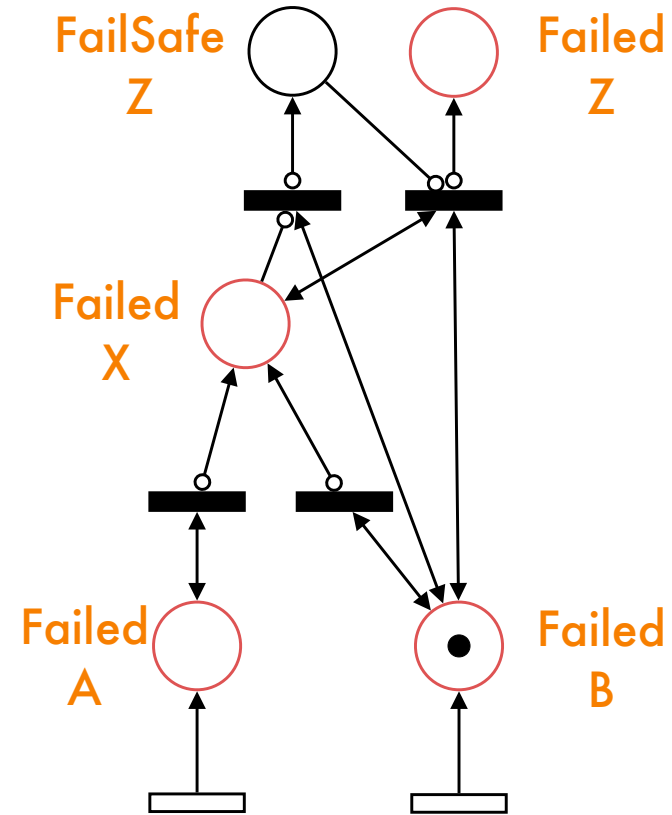


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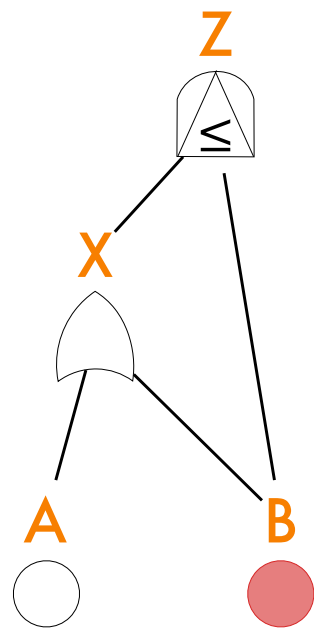


$$\tau_1 = f_B$$

$$\tau_2 = f_B$$

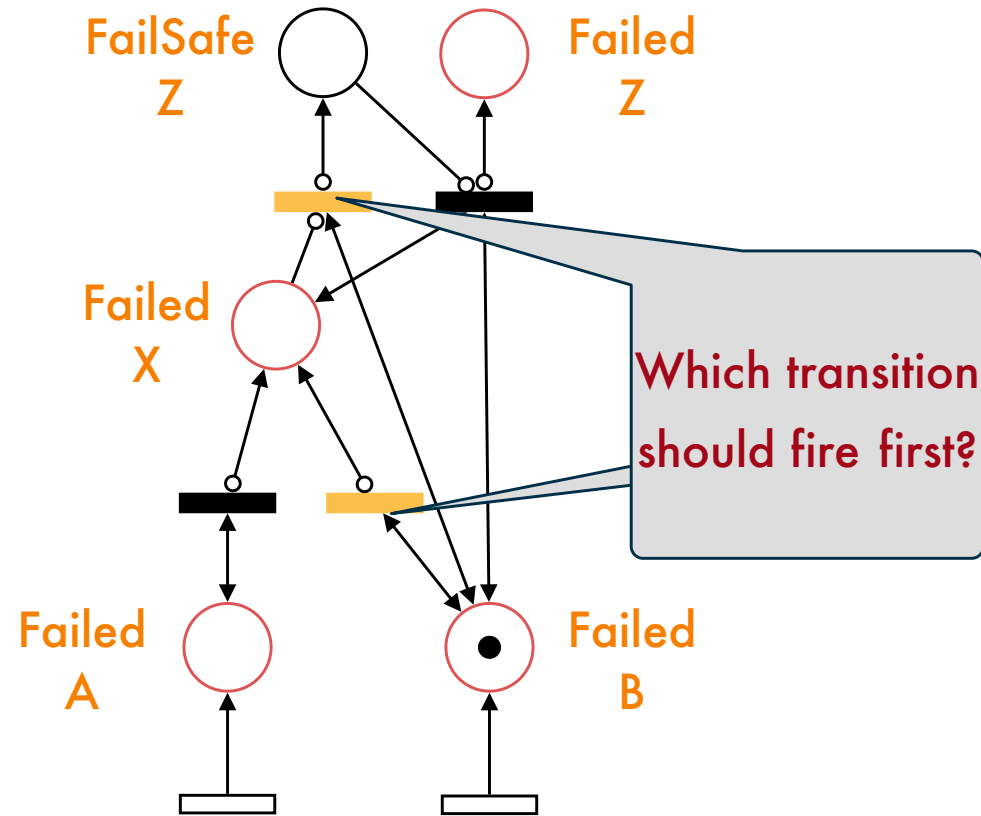


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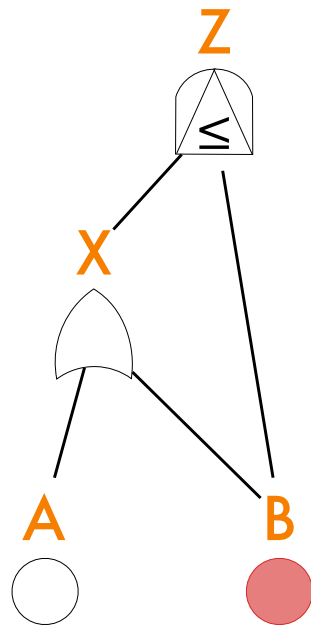


$$\tau_1 = f_B$$

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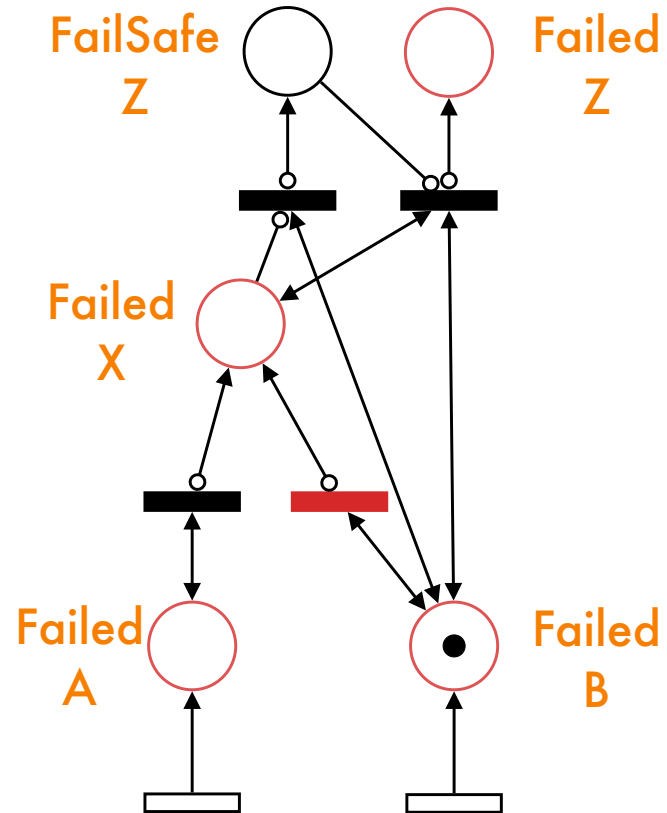


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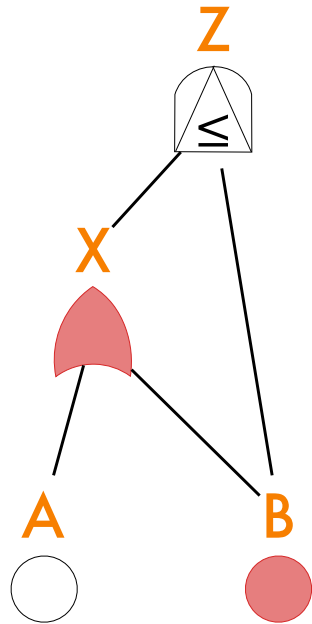


$$\tau_1 = f_B$$

$$\tau_2 = f_B$$

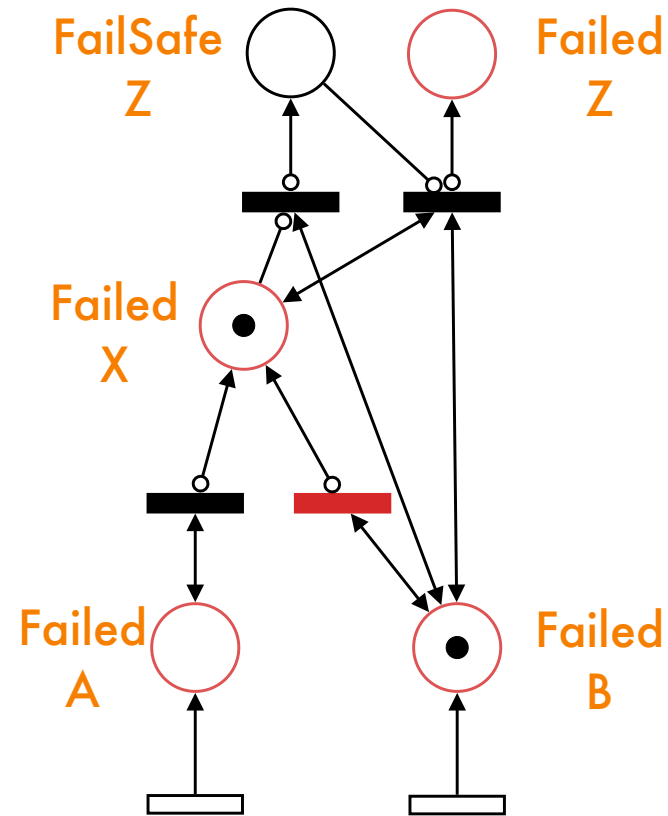


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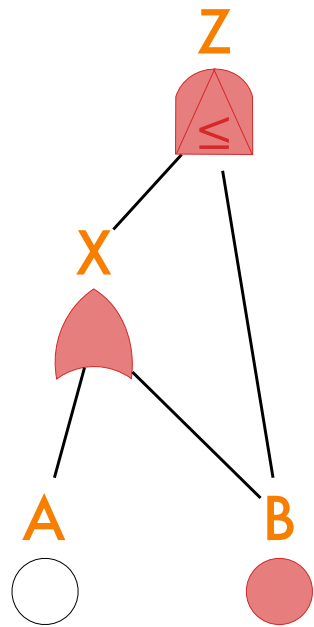
$$\tau_1 = f_B f_X$$

$$\tau_2 = f_B$$



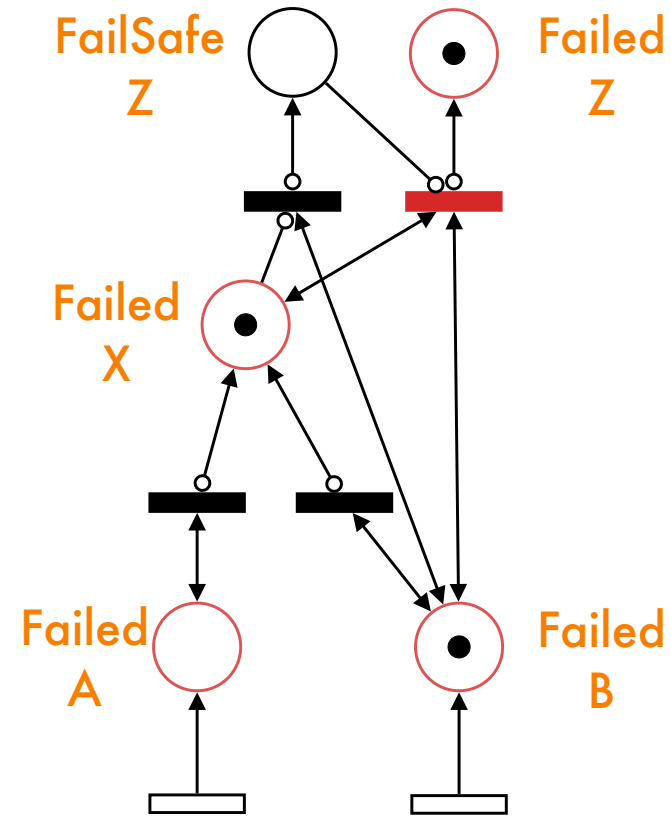


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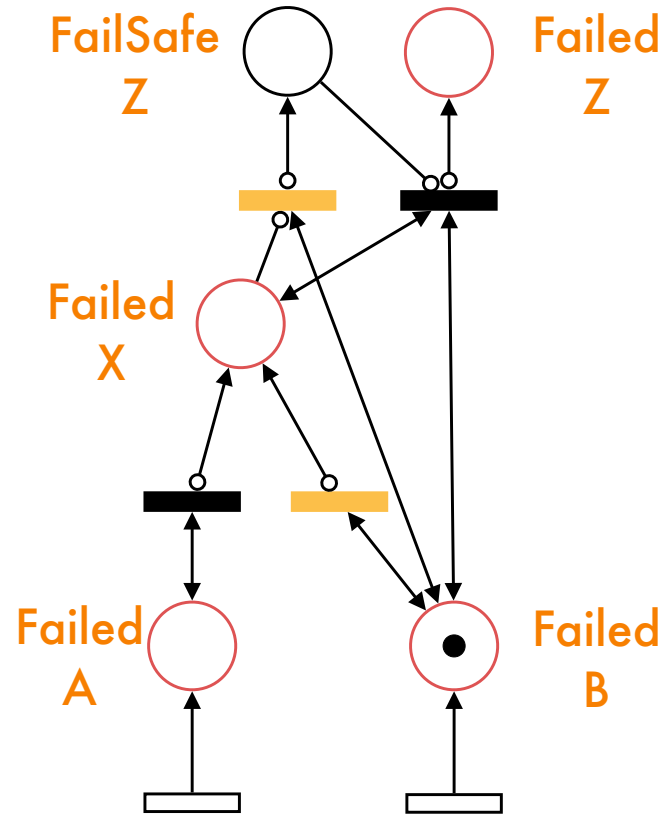
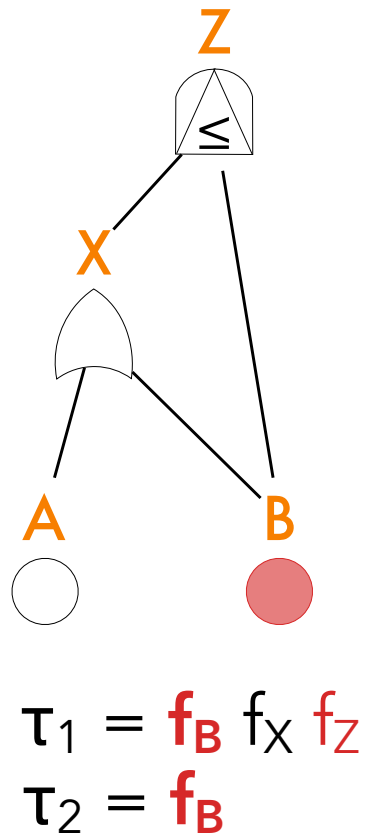


$$\tau_1 = f_B f_X f_Z$$

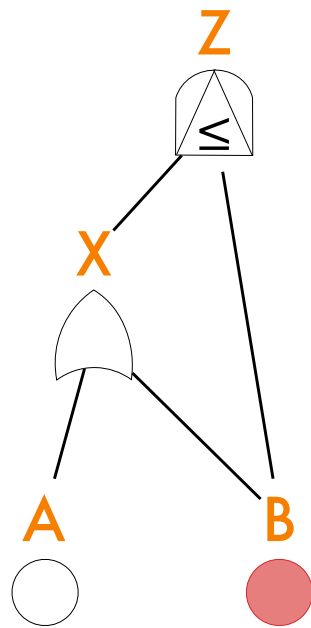
$$\tau_2 = f_B$$



# A Petri Net Approach

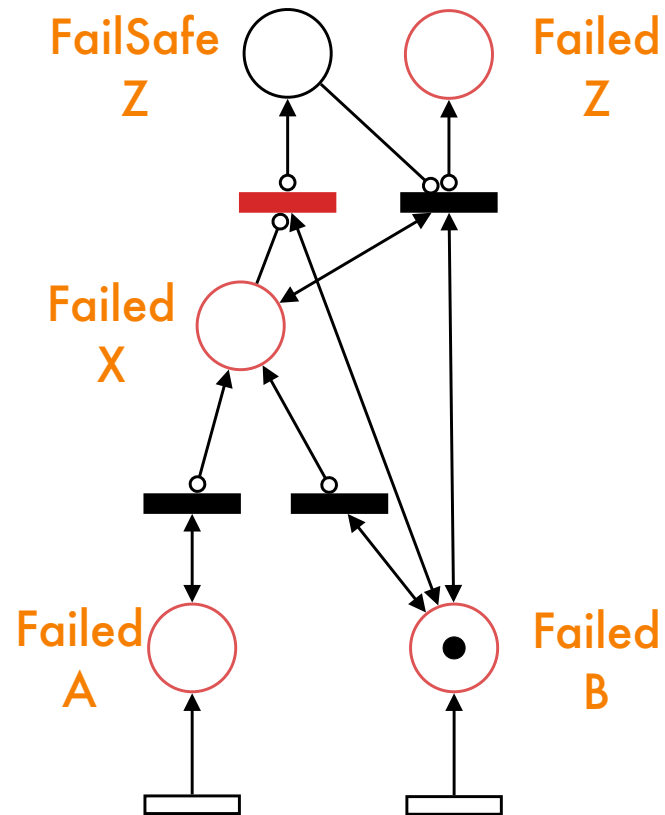


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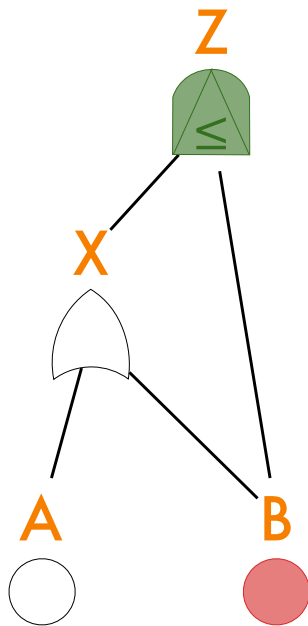


$$\tau_1 = f_B f_X f_Z$$

$$\tau_2 = f_B$$

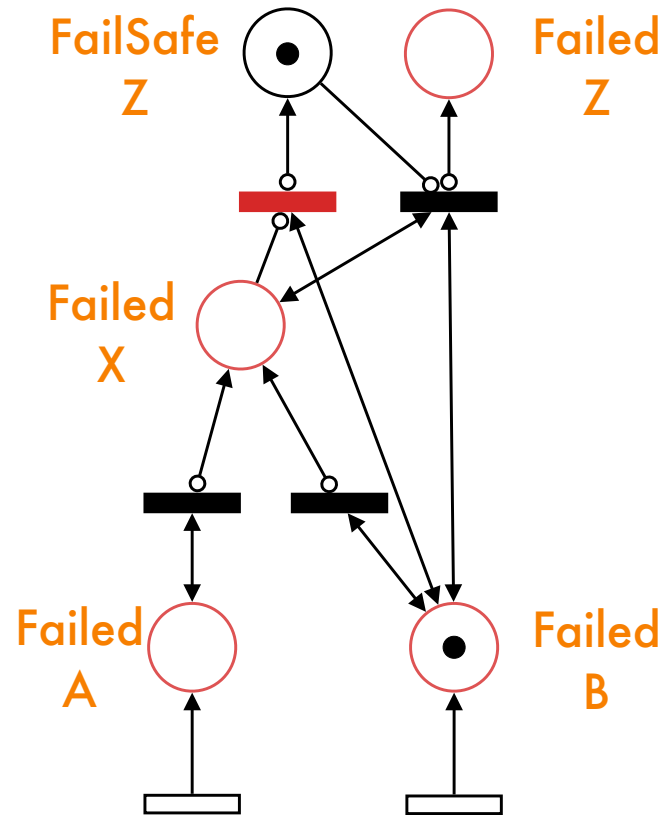


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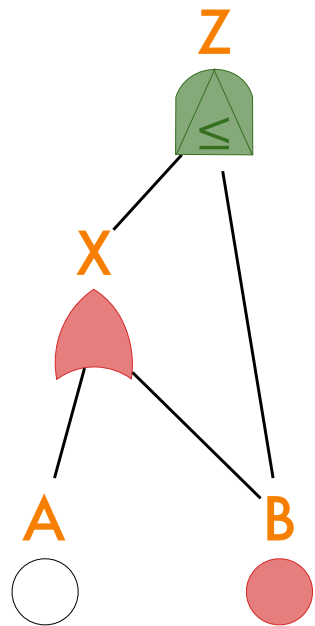


$$\tau_1 = f_B f_X f_Z$$

$$\tau_2 = f_B \textcircled{Z}$$

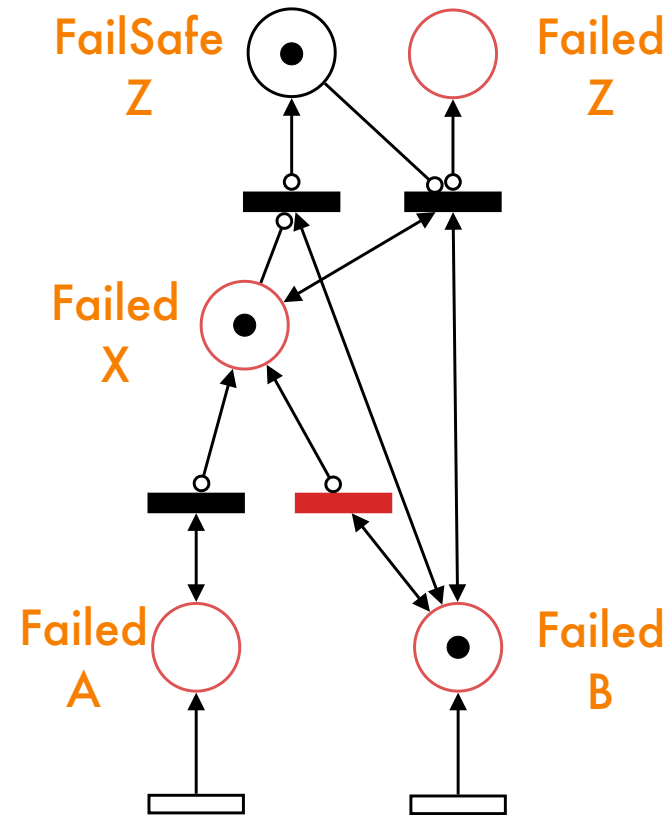


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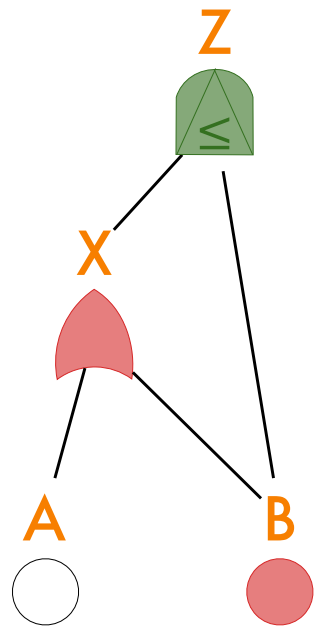


$$\tau_1 = f_B f_X f_Z$$

$$\tau_2 = f_B \ominus_Z f_X$$

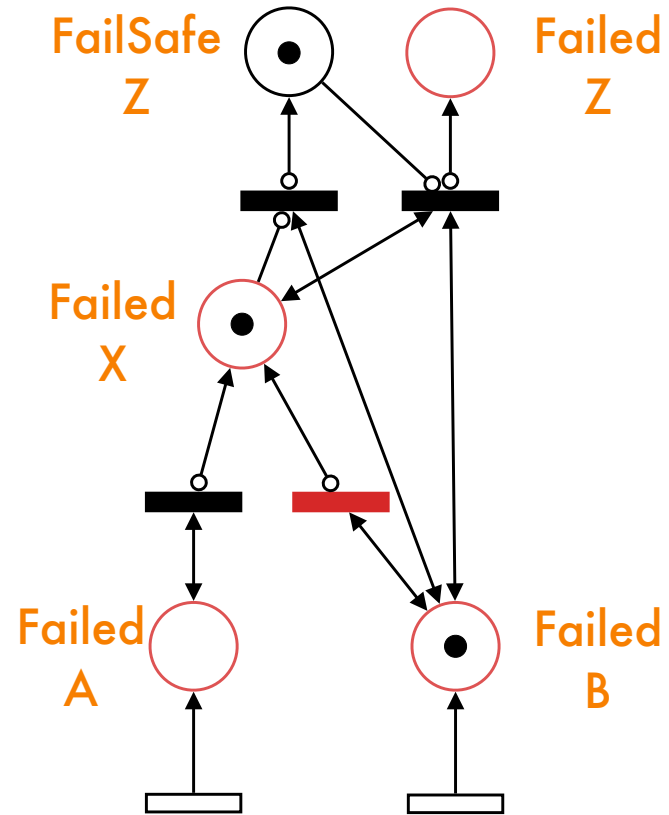


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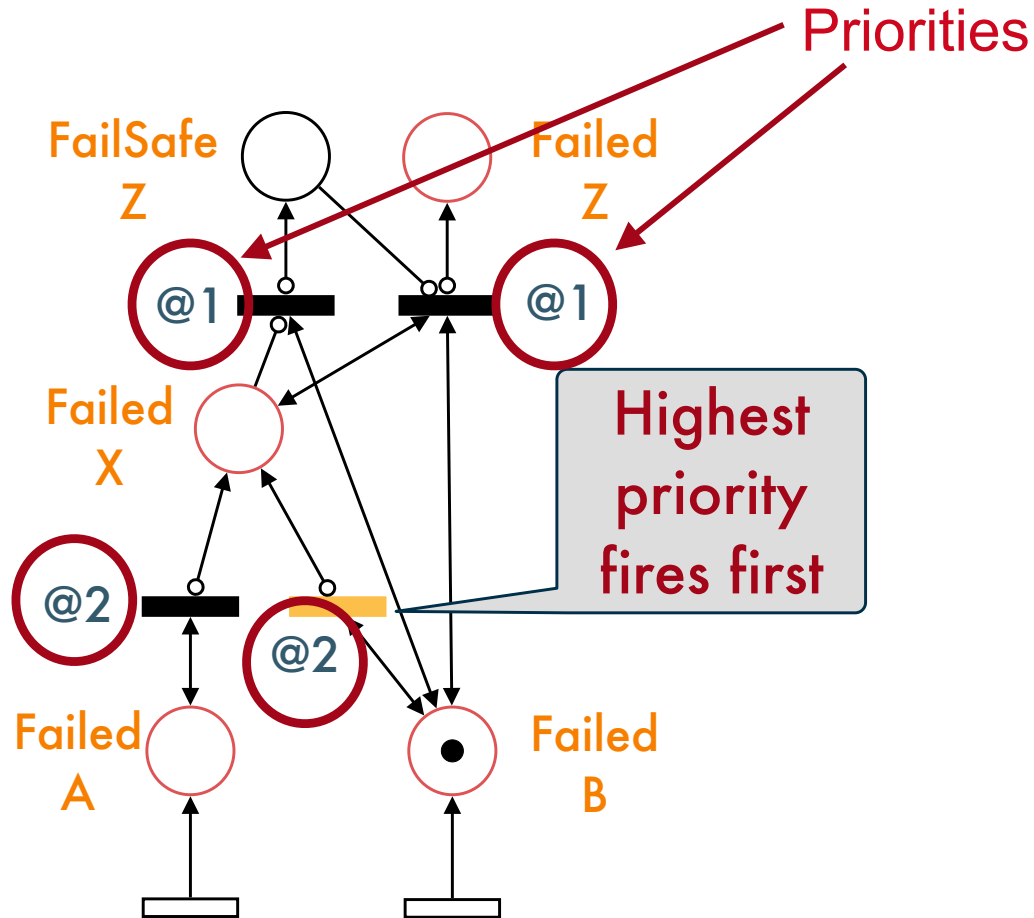
$$\tau_1 = f_B f_X f_Z$$

$$\tau_2 = f_B \textcircled{Z} f_X \textcircled{Z}$$

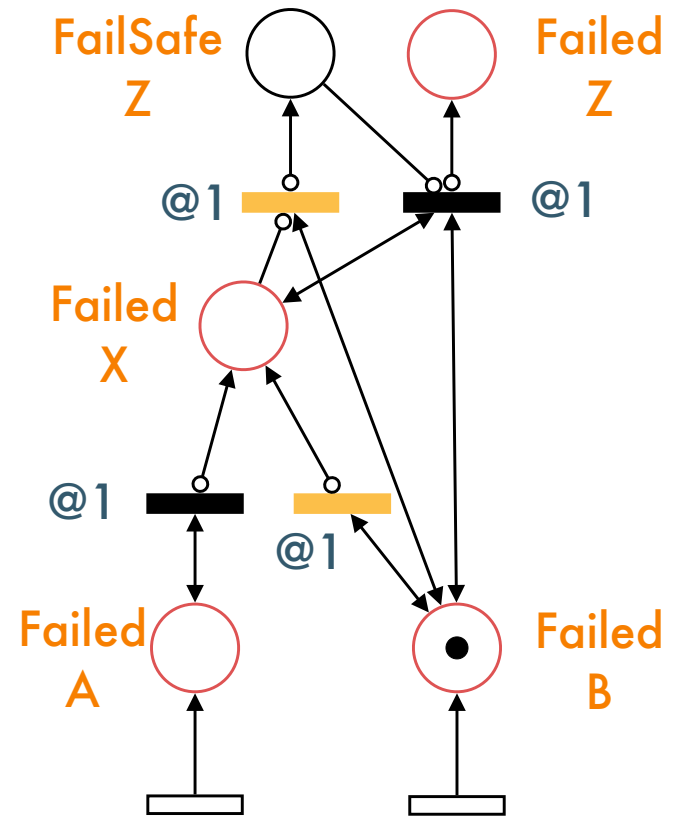


# Priorities in Petri Nets

Storm



DFTCalc



Petri net framework allows to pinpoint semantic differences

# Petri Net Semantics Wrap-Up

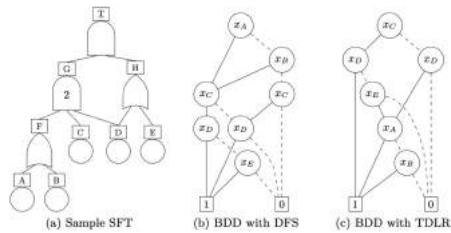
---

- Compositional mapping of DFTs onto GSPNs
- Correctness
  - net semantics is equivalent to (intuitive) event trace semantics
- Petri net properties
  - the size of the net is linear in the size of the DFT
  - the resulting nets are bounded
- Our Petri net framework covers all existing DFT semantics
  - differences are in the priority assignment
  - spare races are non-deterministic or probabilistic

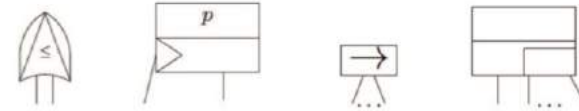


# Talk Overview

1.



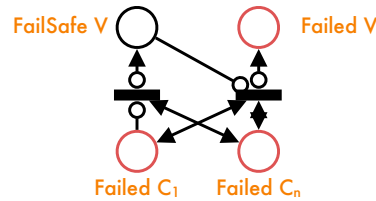
2.



## Dynamic Fault Trees

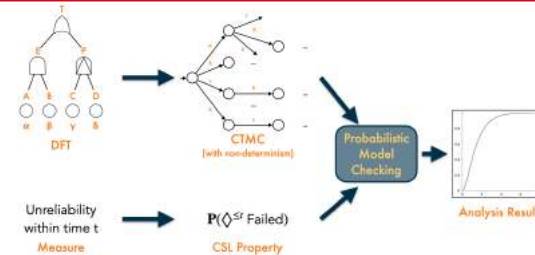
## Classical Static Fault Trees

3.



## Petri Net Semantics

4.



## Scaling Up DFT Analysis

5.



## Industrial Case Studies

6.



## Commercialisation

# Myths About Dynamic Fault Trees

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“Although DFTs are powerful in modeling systems with dynamic failure behaviors, their quantitative analyses are pretty much troublesome, especially for large scale and complex DFTs.”

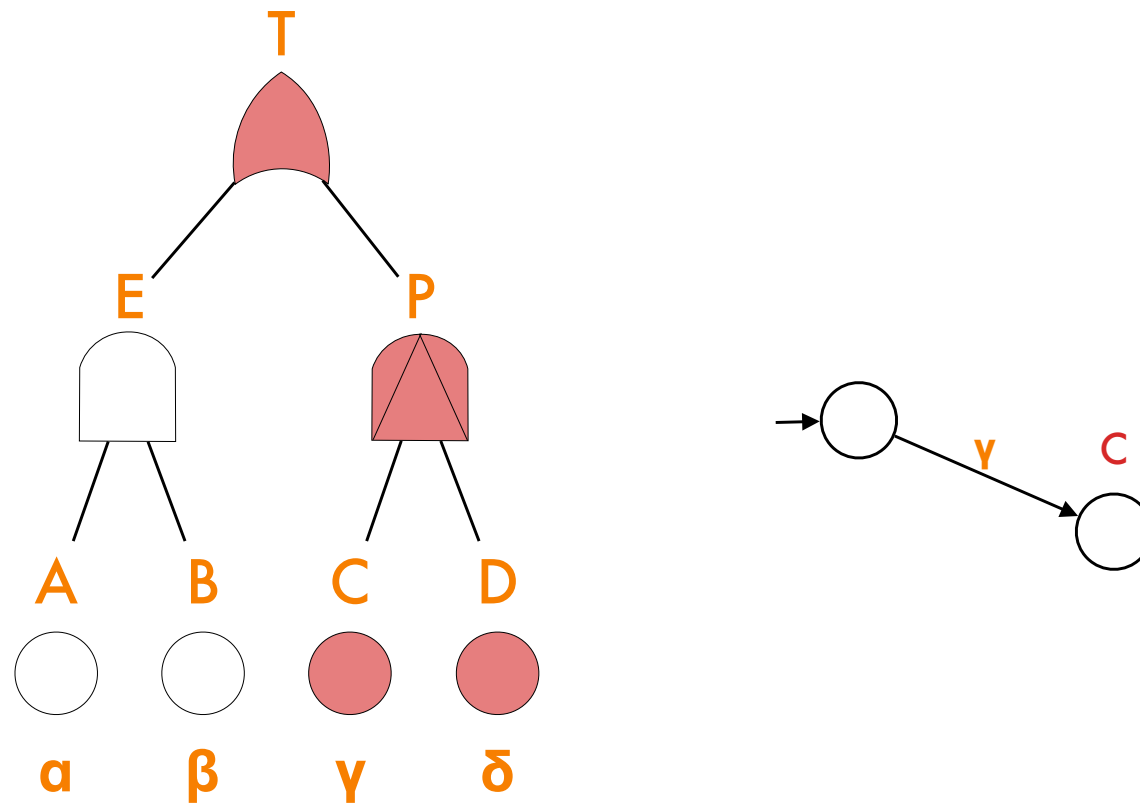
[Ge *et al.*, Rel. Eng. Syst. Safe, 2015]

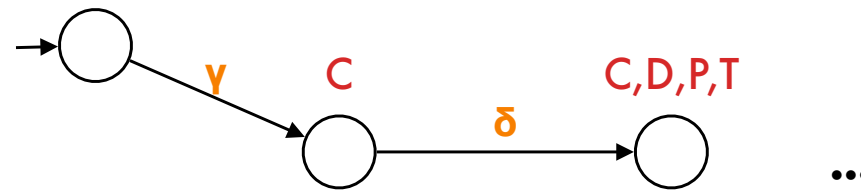
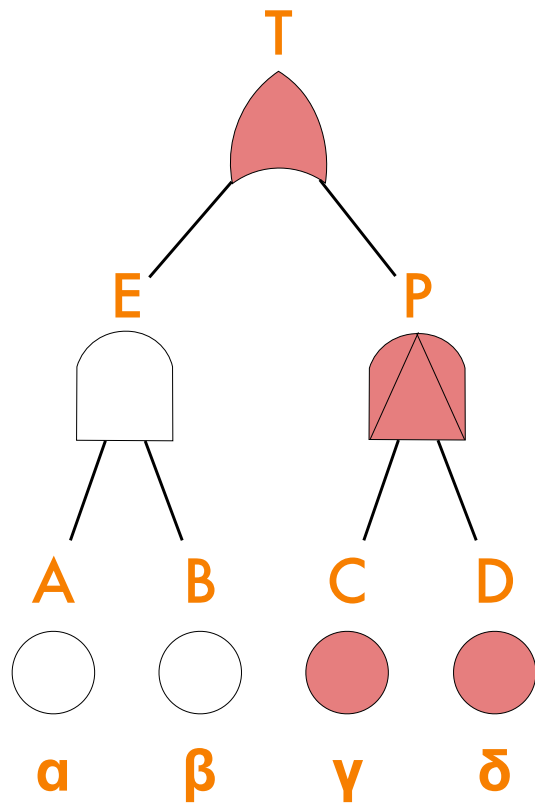
“Although many extensions of fault trees have been proposed, they suffer from a variety of shortcomings. In particular, even where software tool support exists, these analyses require a lot of manual effort.”

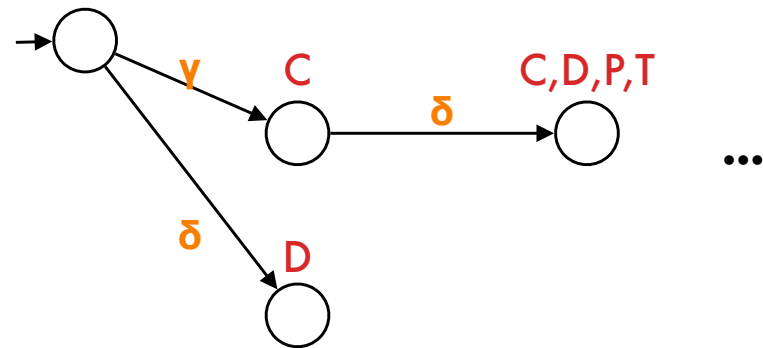
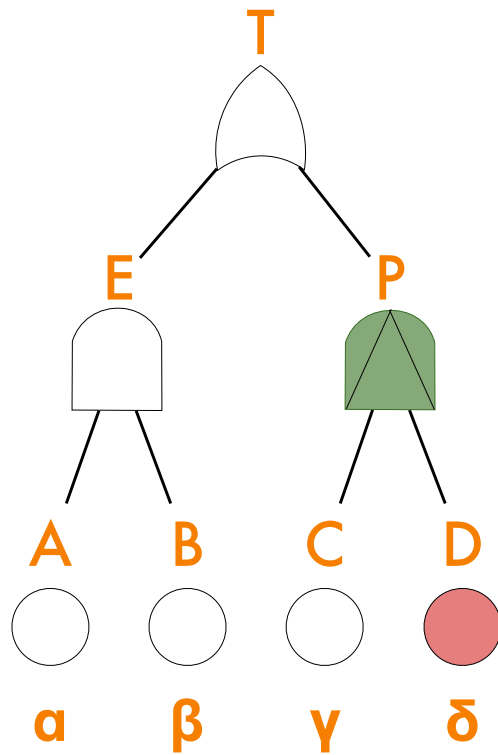
[Kabir, Expert Syst. Appl., 2017]

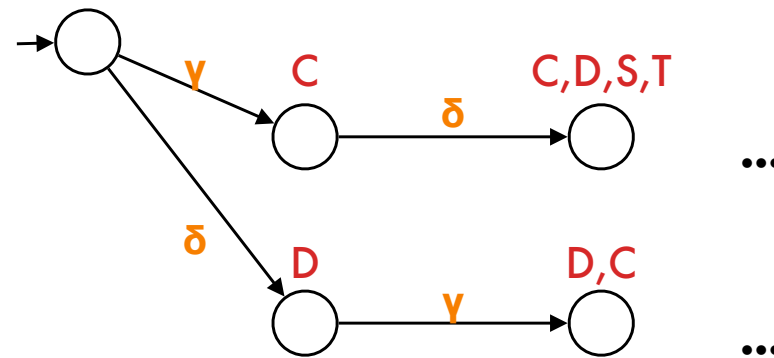
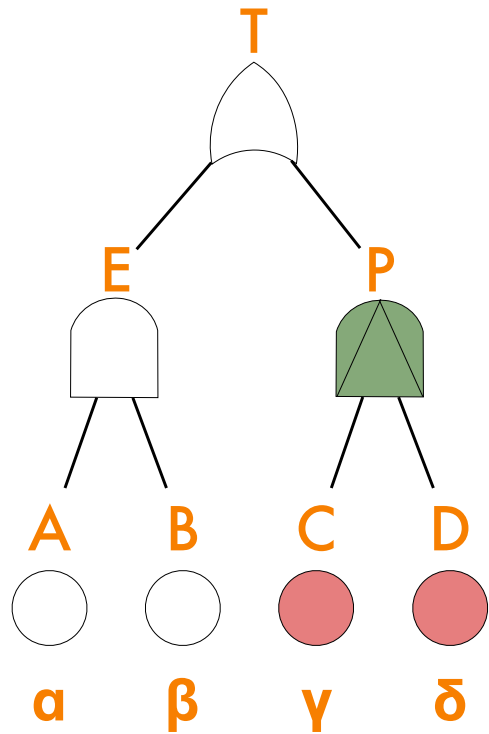
These are all myths. **Scalable and fully automated** DFT analysis is possible.

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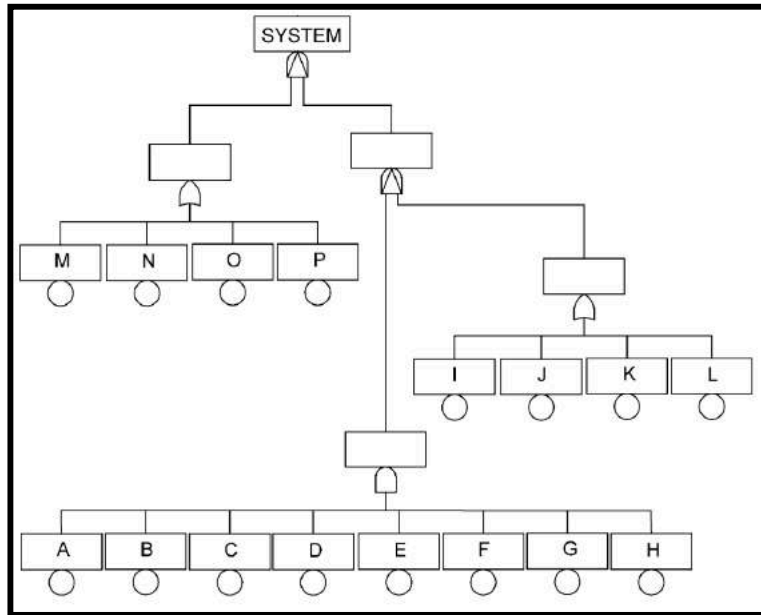








# State Space Explosion Problem?



„[The example was created to] make the corresponding Markov chain of this tree drastically large and practically impossible to solve without resorting to simplifying assumptions and/or approximations“

[Boudali & Dugan 2005]

## Fictitious system DFT



### Naive state-space generation

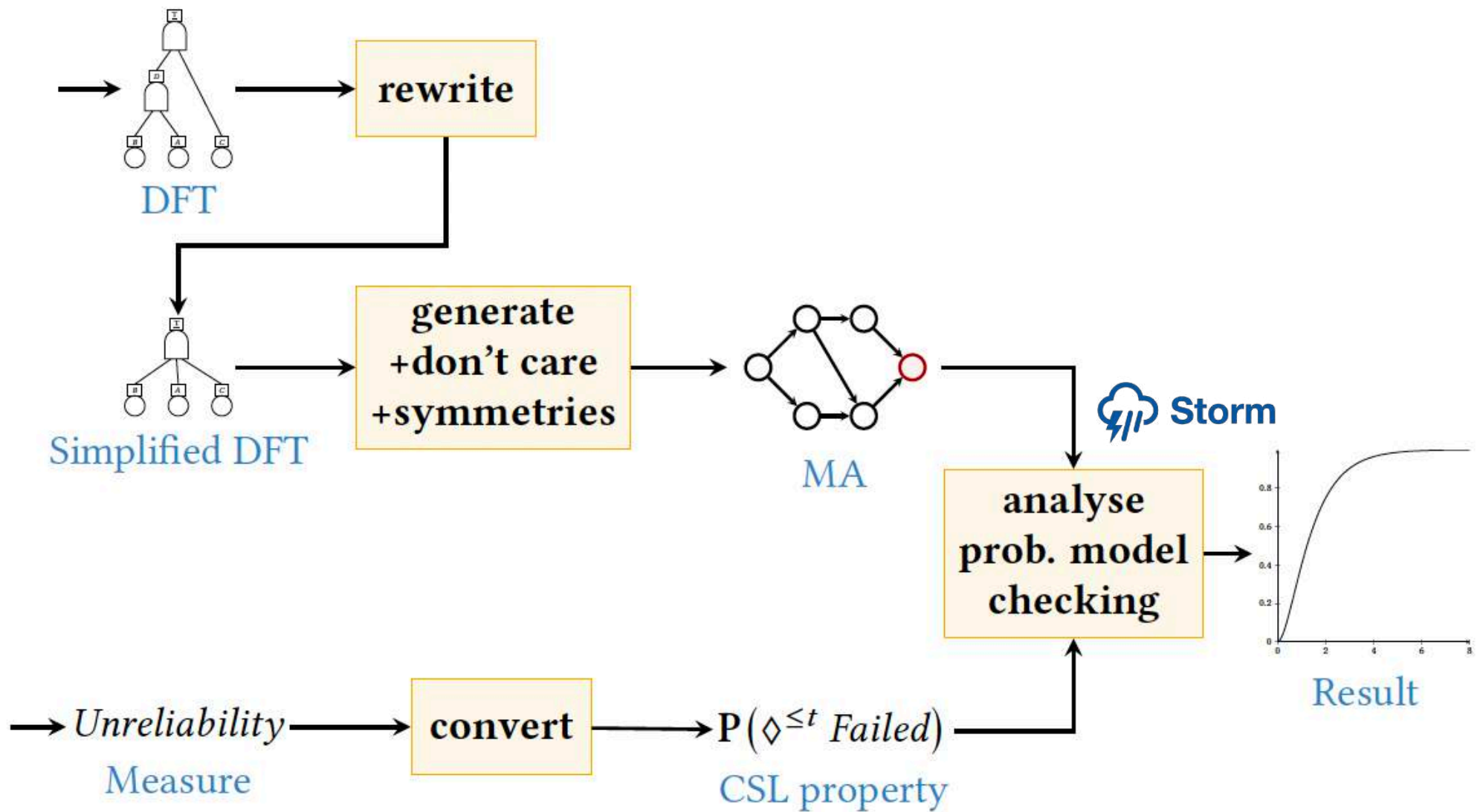
- ✓ 66,001 states
- ✓ Analysis in 1.073 seconds

### Optimised state-space generation

- ✓ 514 states
- ✓ Analysis in 0.015 seconds

Exact result

# What's The Secret?

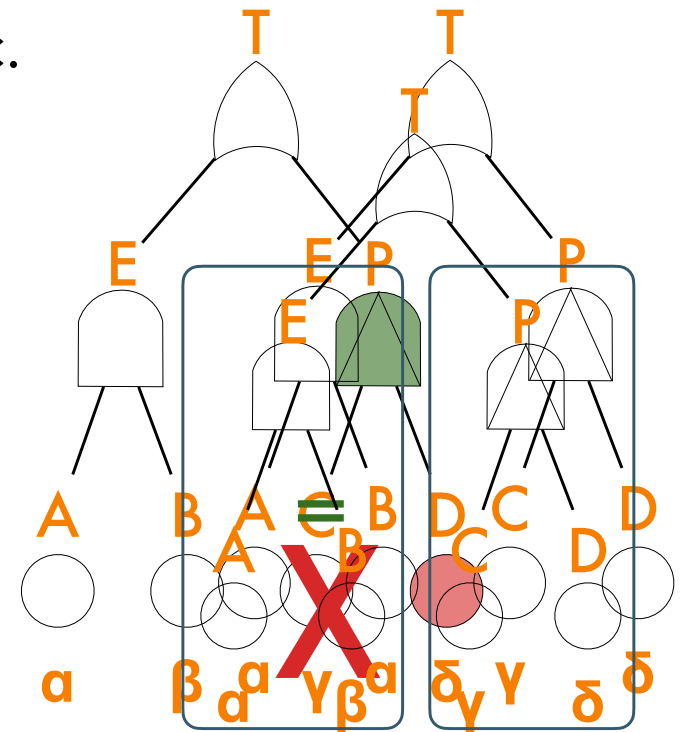


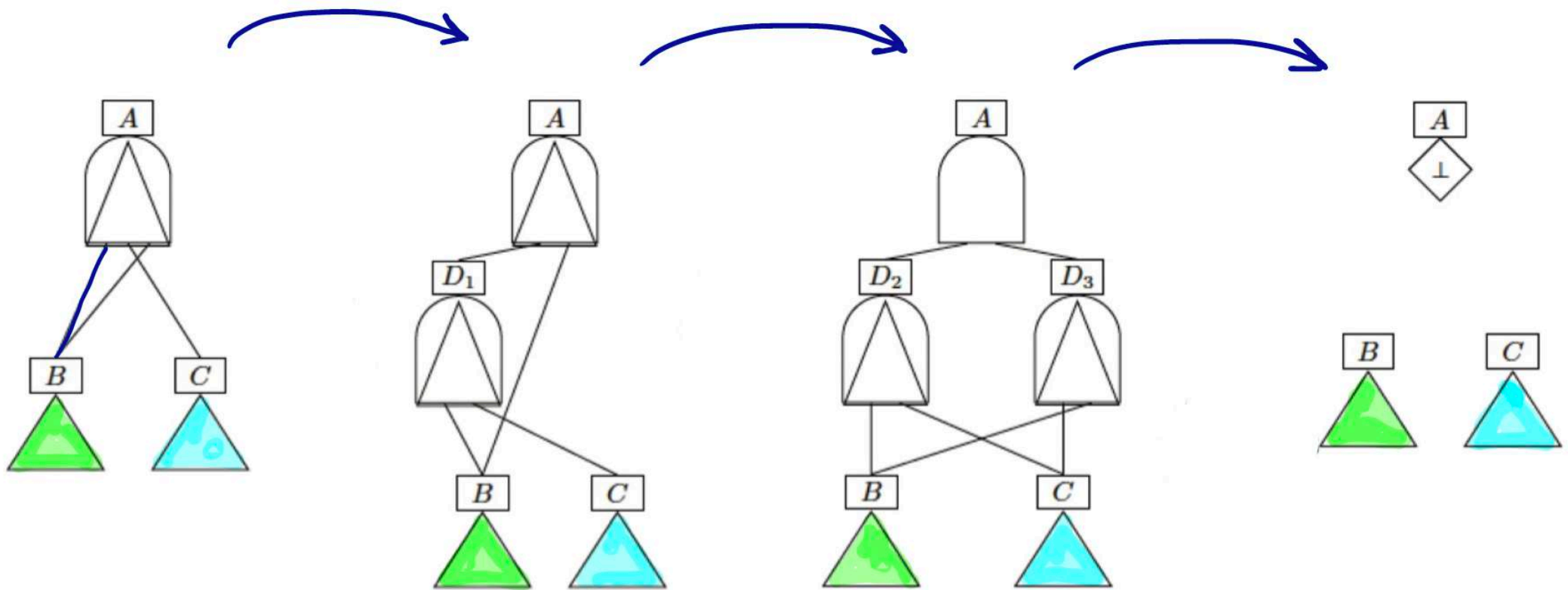


# Optimisations

All these techniques were revised, improved and extended.

- **Don't Care** [Bouissou, Bon, 2003] for BDMP, [Yevkin, 2016]
  - exact status of element is irrelevant for further analysis
  - e.g., fail-safe, completely failed, etc.
- **Symmetries** [Bobbio, Codetta-Raiteri, 2004]
  - present through redundancies
  - merge states which are symmetric
- **Modularisation** [Gulati, Dugan, 1997]
  - analyse sub-parts independently
- Eliminate **spurious non-determinism**
- **Rewrite (simplify) DFTs** before analysis
- **Partial state-space generation**



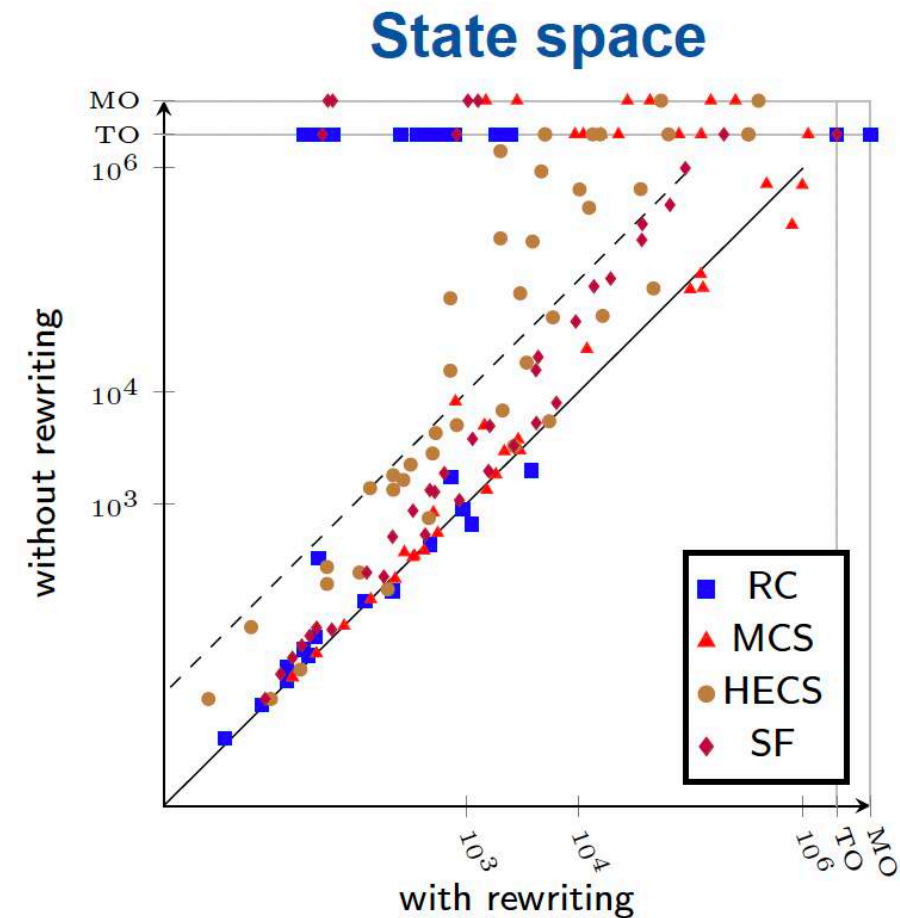
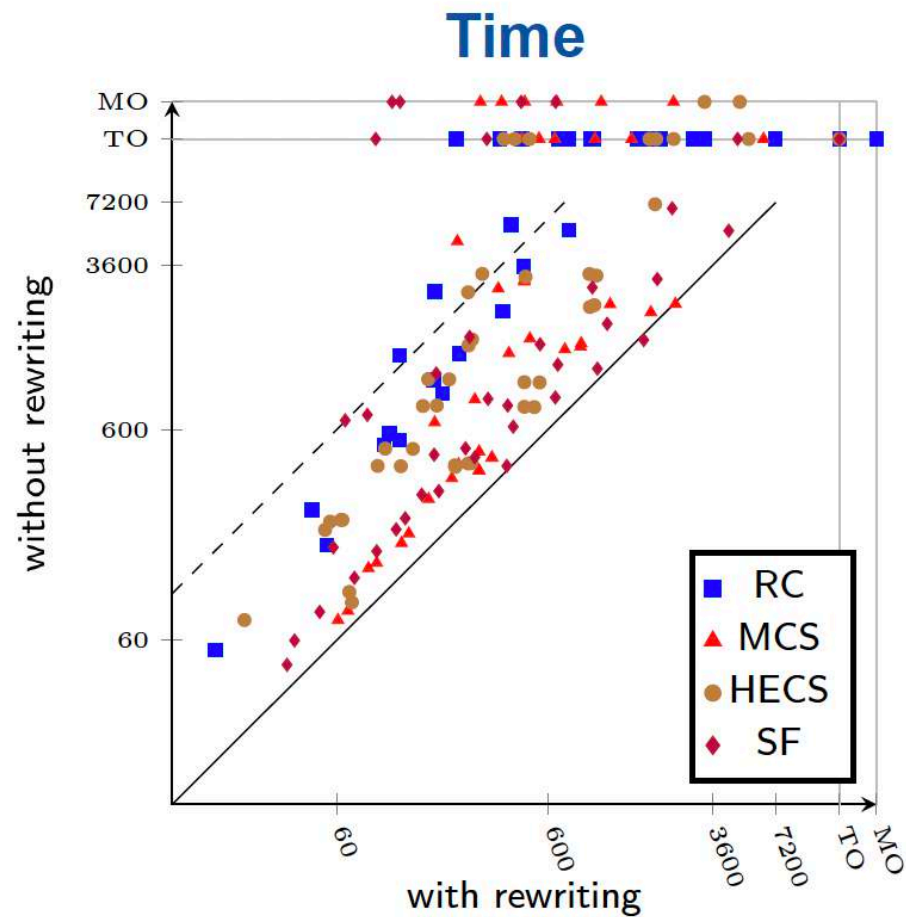


- Context-sensitive rewrite rules
- Interpreted as graph rewriting rules
- Catalogue of 29 rewrite rules
  - flattening of AND, OR and PAND
  - removal of conflicting PAND gates
  - pushing up OR and AND gates
- **Correctness** [Elderhalli et al., SEFM 2019]
  - 22 rules were proven correct using HOL4
  - 1,500 lines of code and about 80 hours effort
  - no formalisation of SPARE and FDEP

# Experimental evaluation

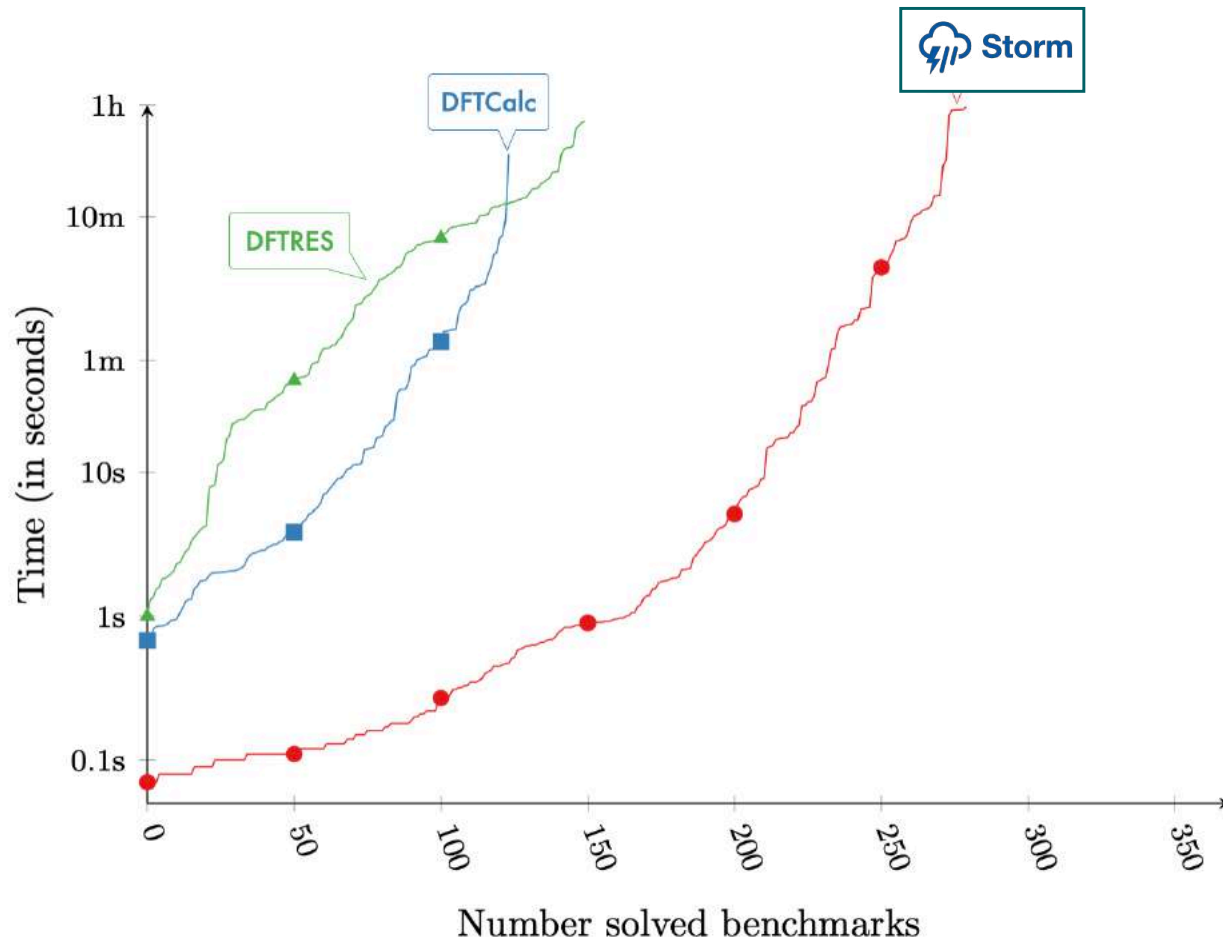


- Analysis with DFTCalc *inside*
- could solve 27% more examples with rewriting



# Evaluation: DFT Analysis Times

<https://dftbenchmarks.utwente.nl/>

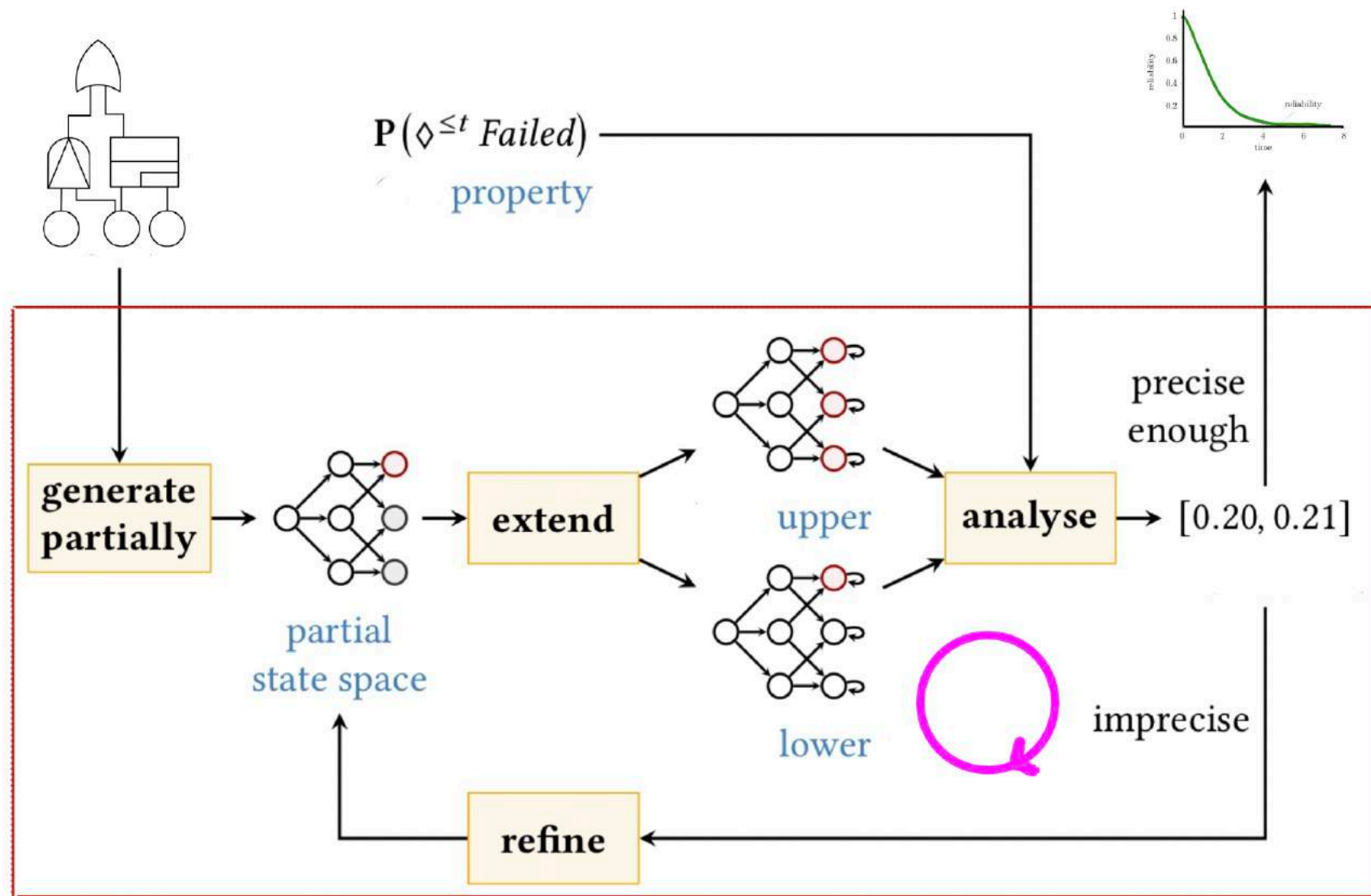


- ✓ Public FFORT benchmark suite
- ✓ Unreliability and MTTF
- ✓ 369 benchmarks
- ✓ Comparison to
  - ✓ DFTRes (2020, simulation)
  - ✓ DFTCalc (2013, compositional)
- ✓ 2.1 GHz, 16 GB RAM
- ✓ Error bound: **10<sup>-4</sup>**

**Storm solves more benchmarks in 1 second than others in 1 hour**

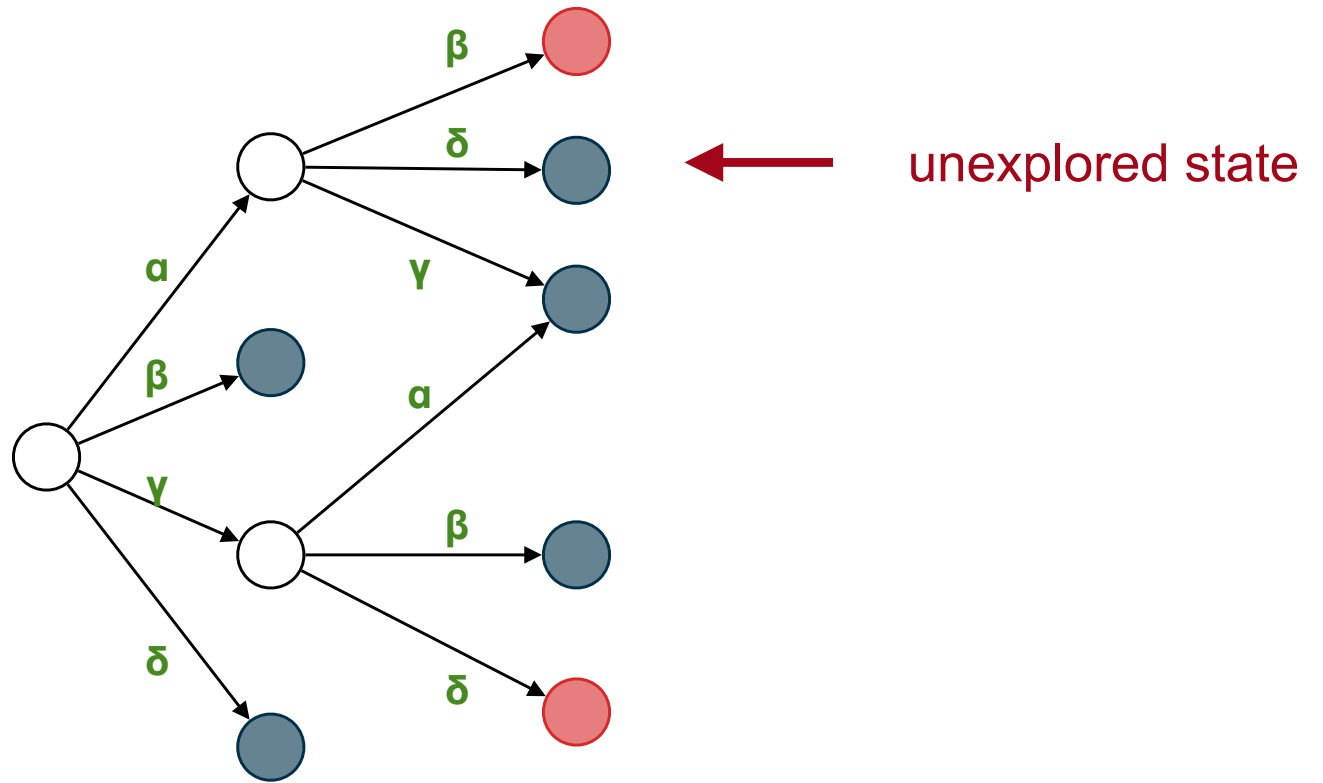
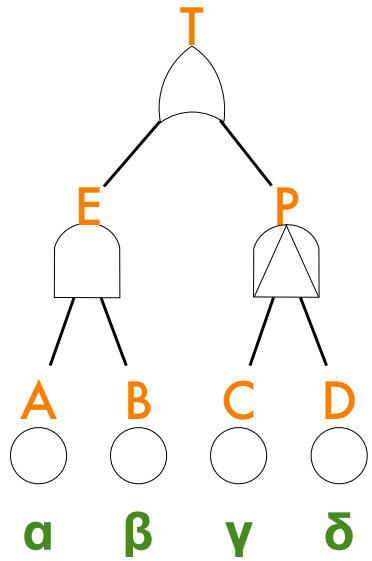
# Partial State-Space Generation

[Volk, Junges, K., IEEE TII 2018]

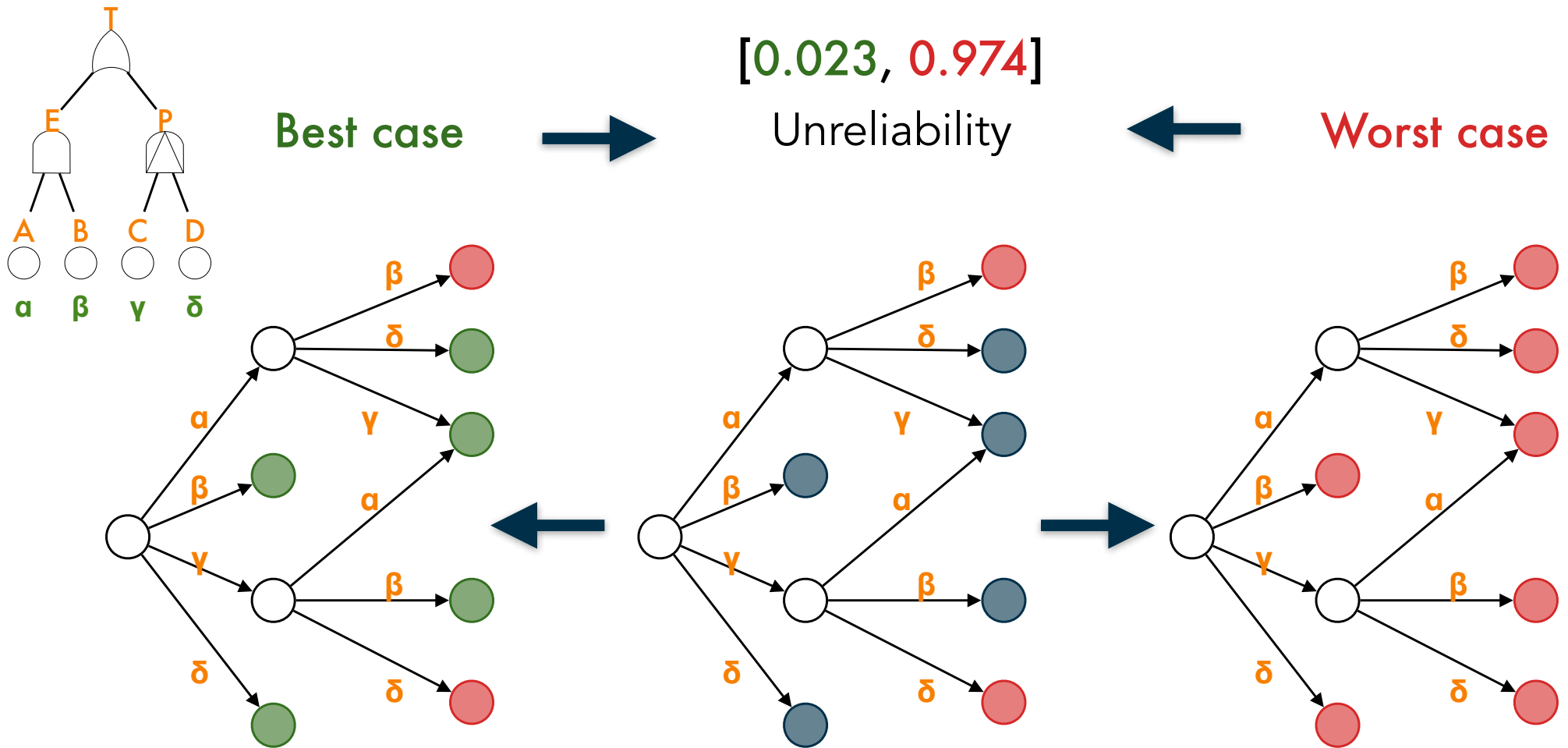


# Partial State-Space Generation

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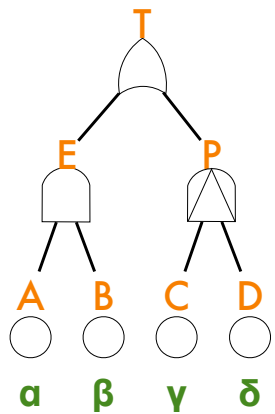


# Partial State-Space Generation

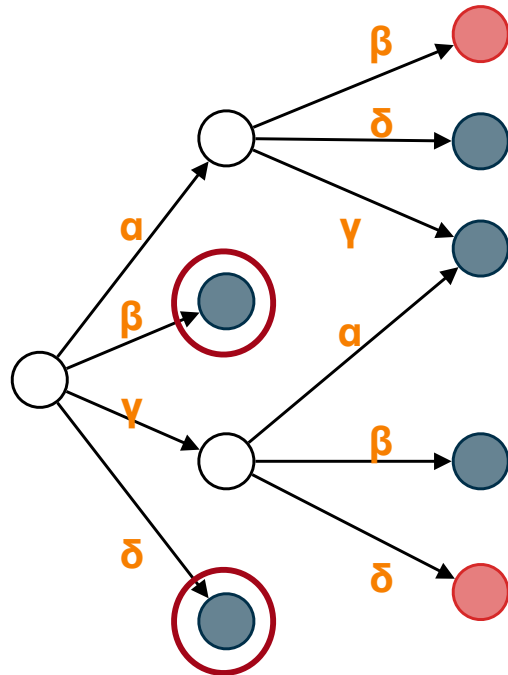




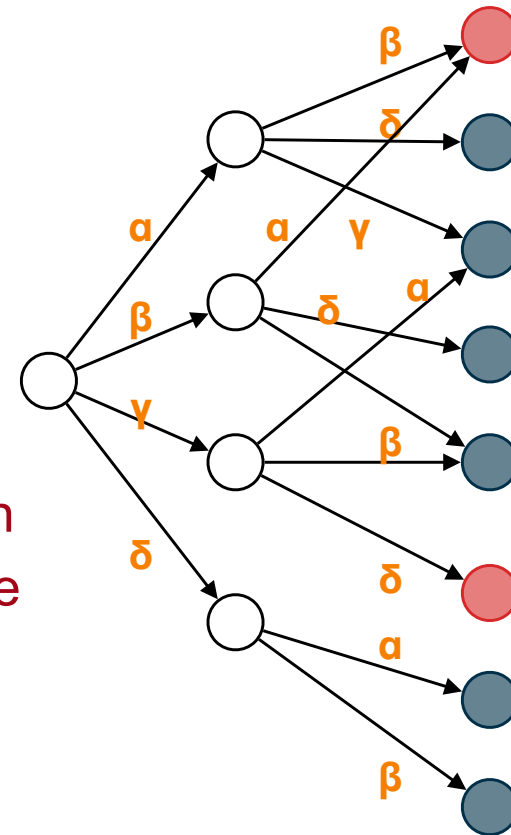
# Partial State-Space Generation



**[0.023, 0.974]**  
Unreliability

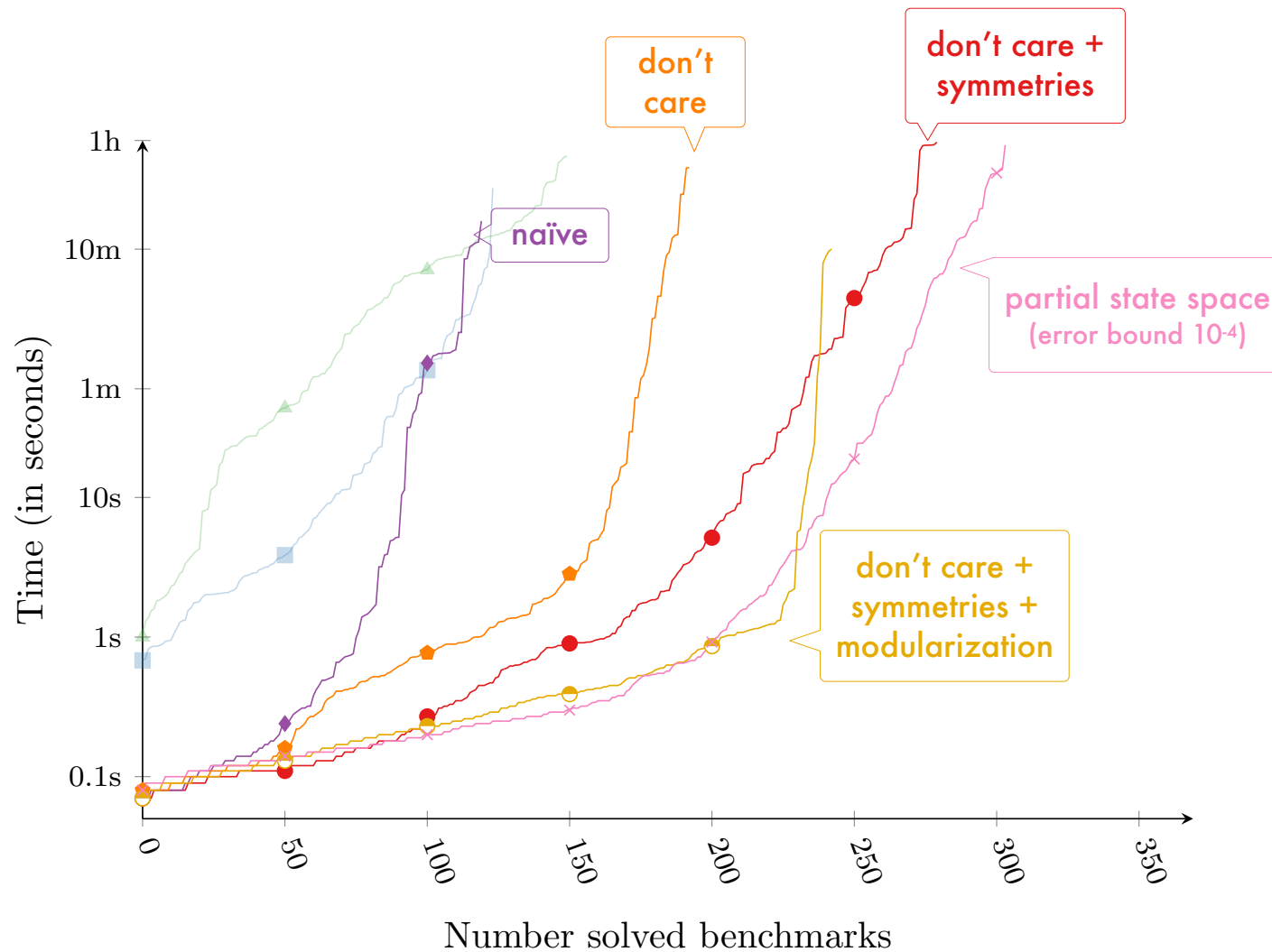


**[0.145, 0.535]**  
Unreliability



Heuristics which states to explore

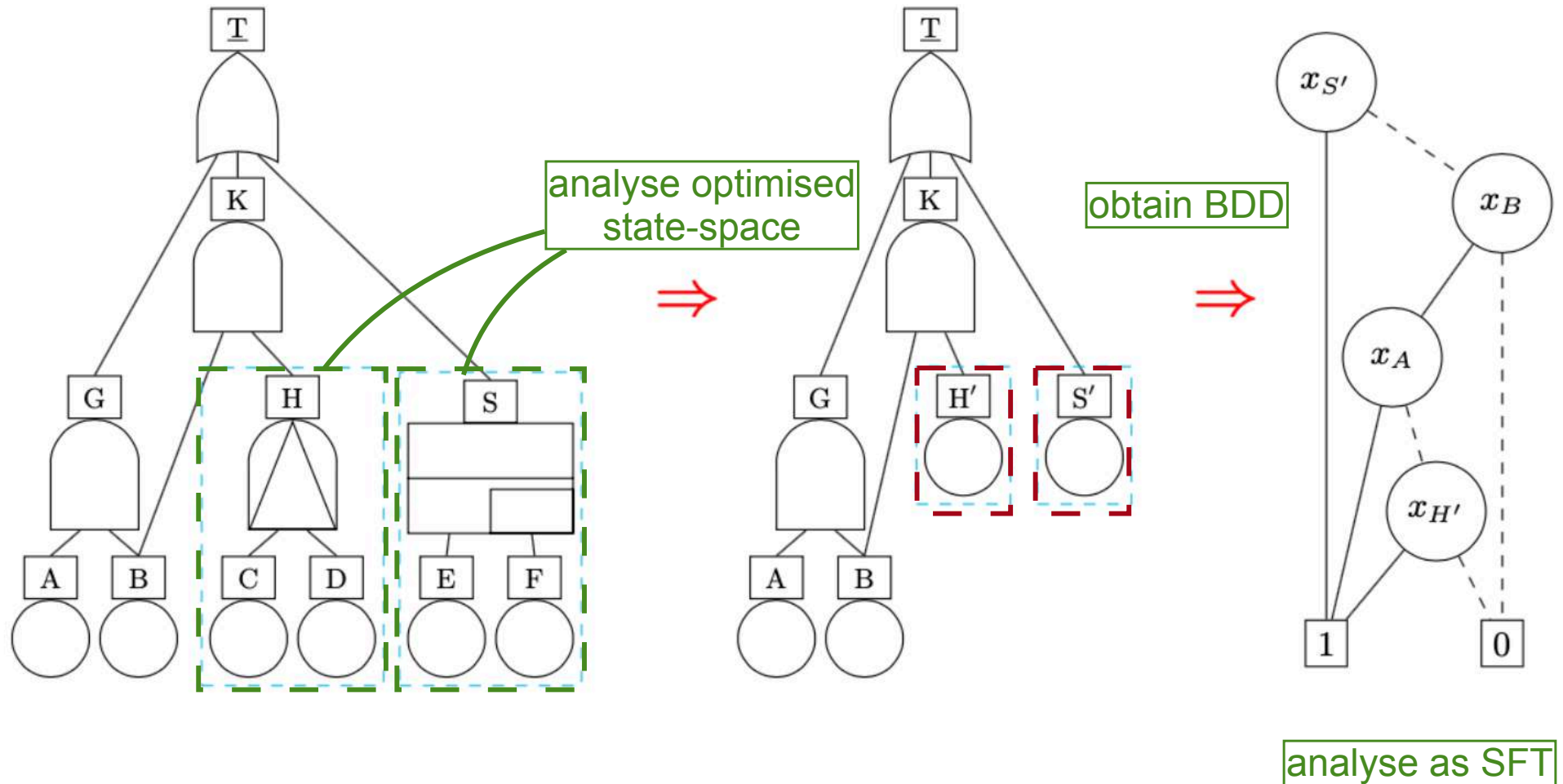
# The Effect of the Optimisations



**The combination of all optimization techniques pays off**

# What If DFTs Contain Large Static Parts?

[Basgöze et al., NASA FM 2022]



# Experiments: DFTs with Static Parts

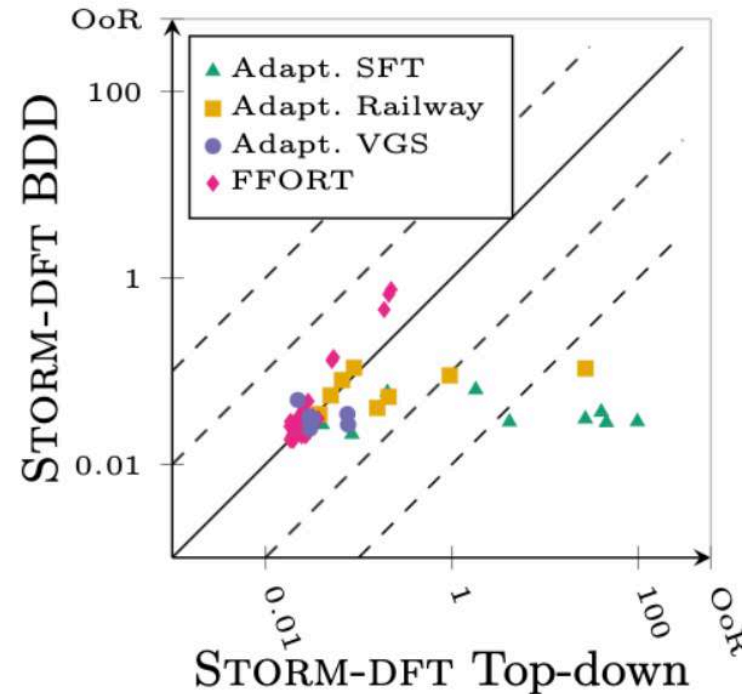
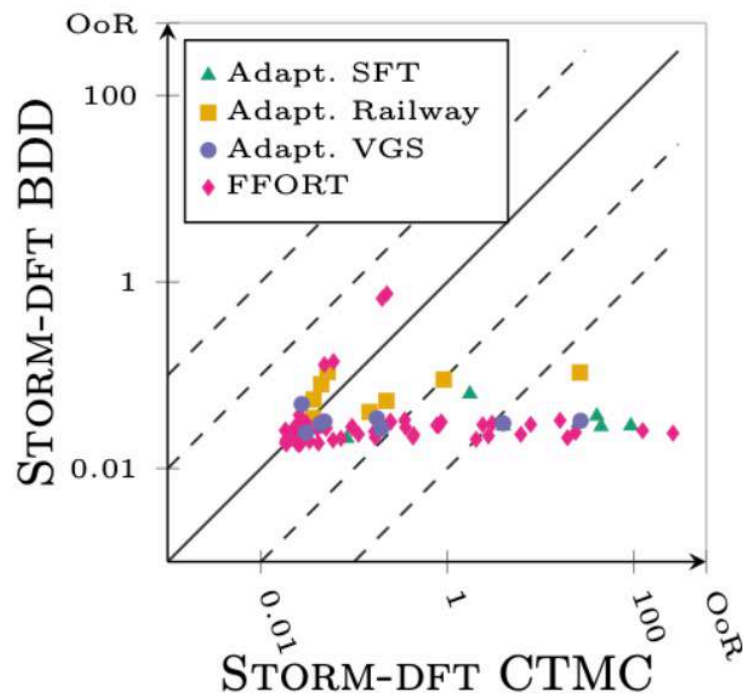
[Basgöze et al., NASA FM 2022]

Benchmark set	#BEs	#Static gates	#Dyn. gates	#BEs mod.	#Static gates mod.
Adapt. SFT	32-1574	26-1628	3	25-1623	21-1623
Adapt. Railway	194-545	153-487	19-54	22-54	40-168
Adapt. VGS	54-99	31-59	6-20	1-79	0-39
FFORT	6-87	1-50	0-44	1-50	0-21

after modularisation



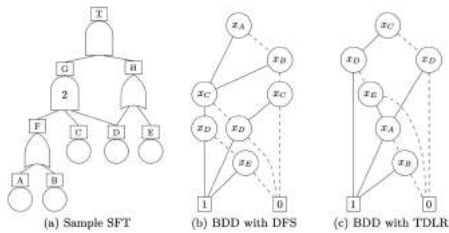
all run times in seconds



Outperforms Markov chain analysis and modularisation

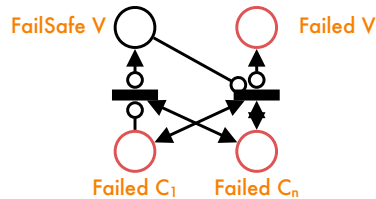
# Talk Overview

1.



## Classical Static Fault Trees

3.



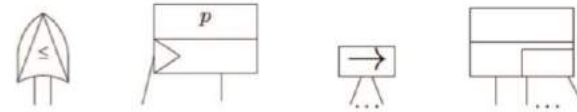
## Petri Net Semantics

5.



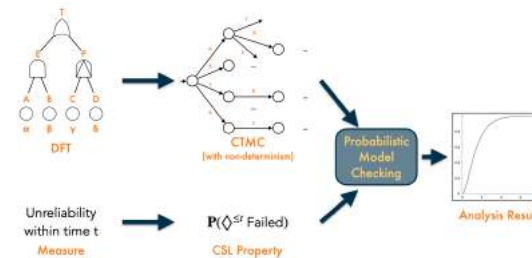
## Industrial Case Studies

2.



## Dynamic Fault Trees

4.



## Scaling Up DFT Analysis

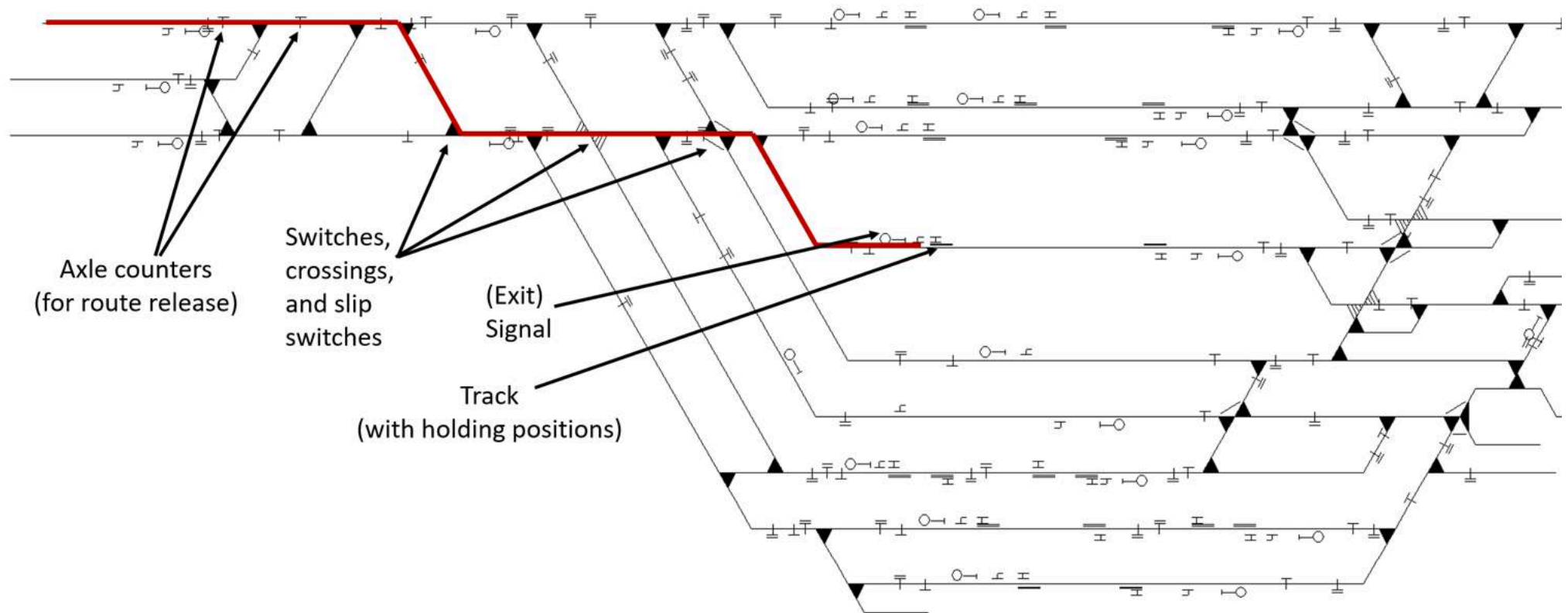
6.



## Commercialisation



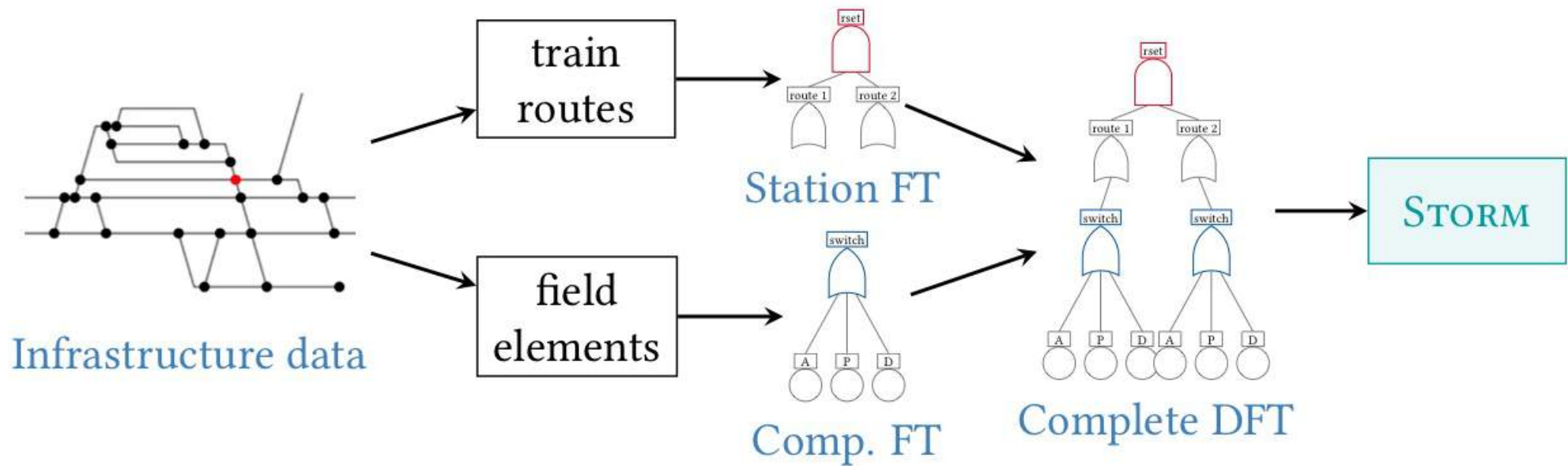
# Train Routability



train path must be set to run train

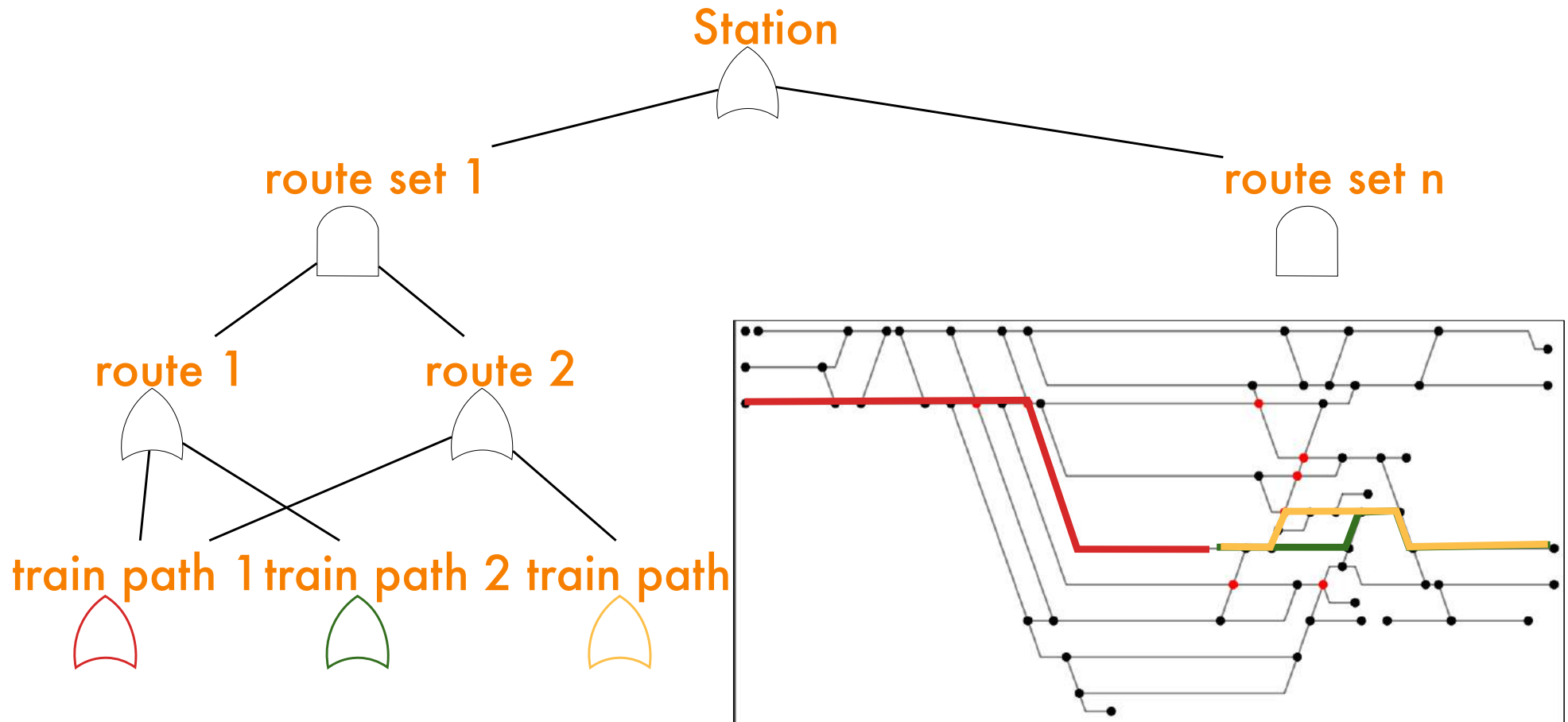
field elements must be operational and in correct position

# Our Approach





# Station Fault Tree



# Field Elements

---

## Other field elements

- ✓ **Slip switch**
  - ✓ modeled as two switches
- ✓ **Crossing**
- ✓ **Track clearance detection**
  - ✓ permanent and transient failure
- ✓ **Signal**

## Failure rates

### ✓ **Switches**

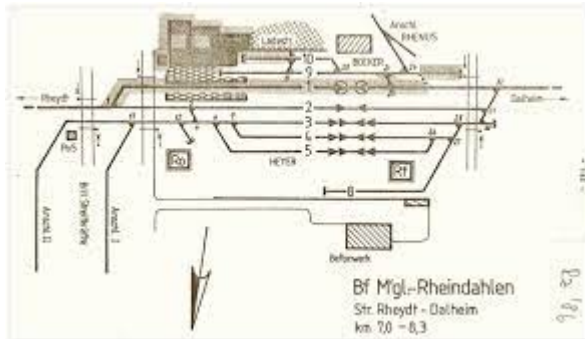
- ✓ data from UK railway network
- ✓ failure types:
  - ✓ Actuation
  - ✓ Control/Power
  - ✓ Detection
  - ✓ Locking
  - ✓ Permanent Way

### ✓ **Other field elements**

- ✓ use data from NL, N, etc.

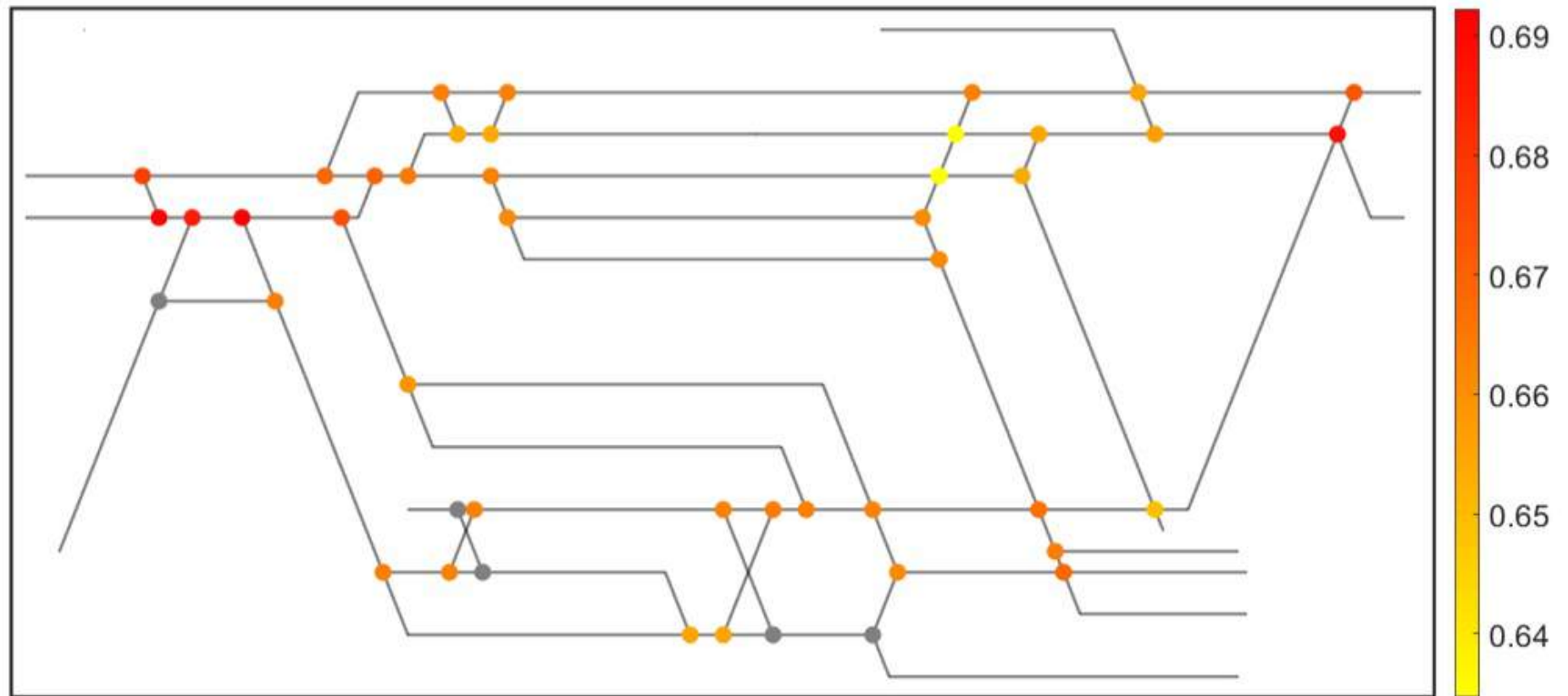
# Criticality Assessment of Railway Infrastructures

Scenario				Railway			
Id	Station	Variant	Max fail	#Route sets	#Routes	#Train paths	#Components
1	Aachen	std	$\infty$	61	61	62	53
2		alt 5	4	23	115	41	54
3	Herzog.	std	$\infty$	11	11	15	22
4		alt 5	4	9	19	15	24
5		alt 5	6	9	19	15	24
6	M'gladb.	std	$\infty$	26	26	32	40
7		alt 5	4	11	43	25	41

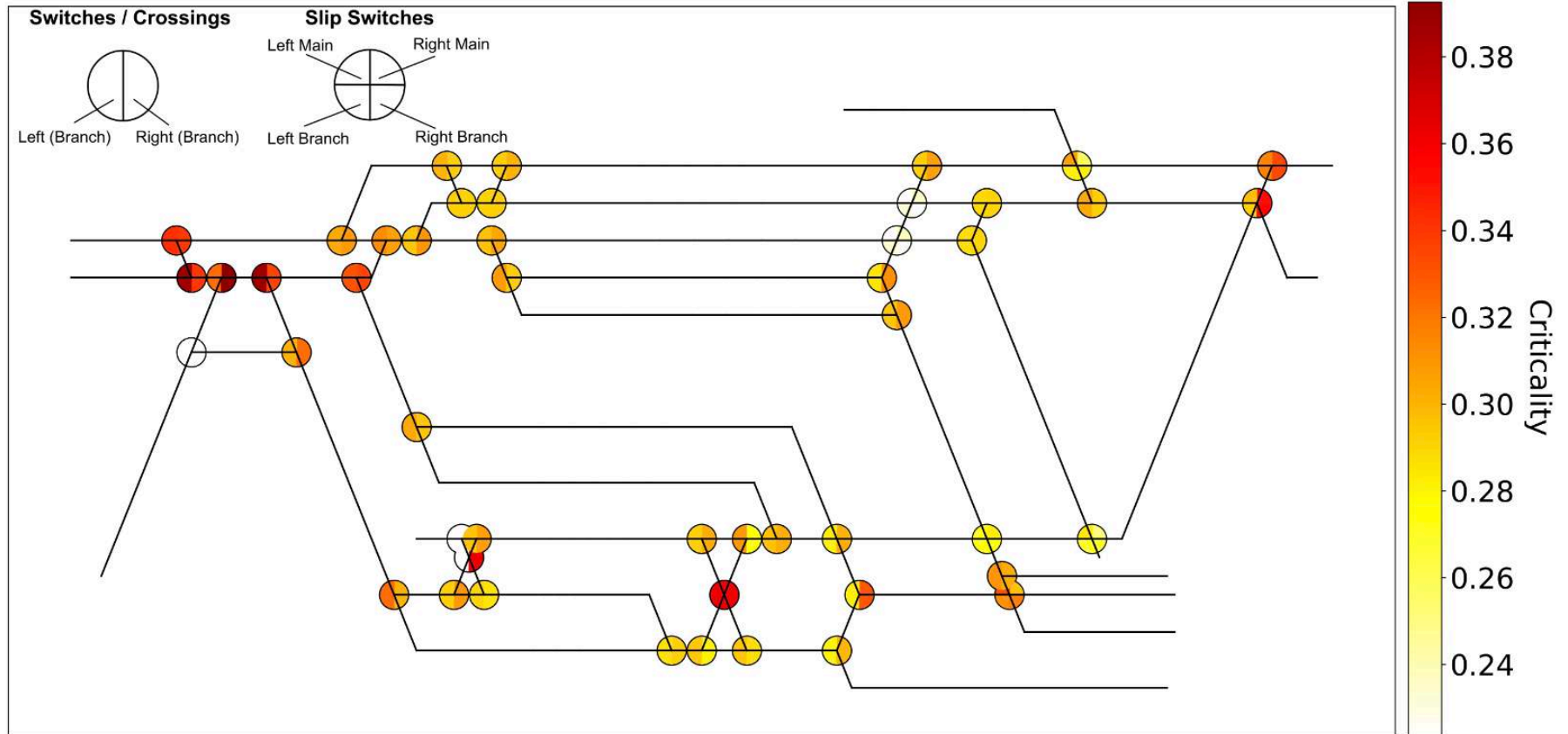


Id	DFT			CTMC		
	#BE	#Static	#Dynamic	#States	#Transitions	Build time [s]
1	544	459	54	2 049	13 313	0.11
2	536	451	53	11 371 990	45 946 651	2 006.16
3	194	137	19	257	1 281	0.04
4	214	153	21	275 073	1 109 037	12.33
5	214	153	21	17 592 280	106 375 167	1 110.48
6	480	325	48	8 193	61 441	27.79
7	490	325	49	6 224 521	24 798 158	645.51

# Criticality Assessment of Railway Station Areas



Criticality of Mönchengladbach Hbf



$$I_v^t = \frac{\partial \text{Unreliability}_{\text{TLE}}^t}{\partial \text{Unreliability}_v^t}$$

Birnbaum importance index for switch branches  
Mönchengladbach Hbf

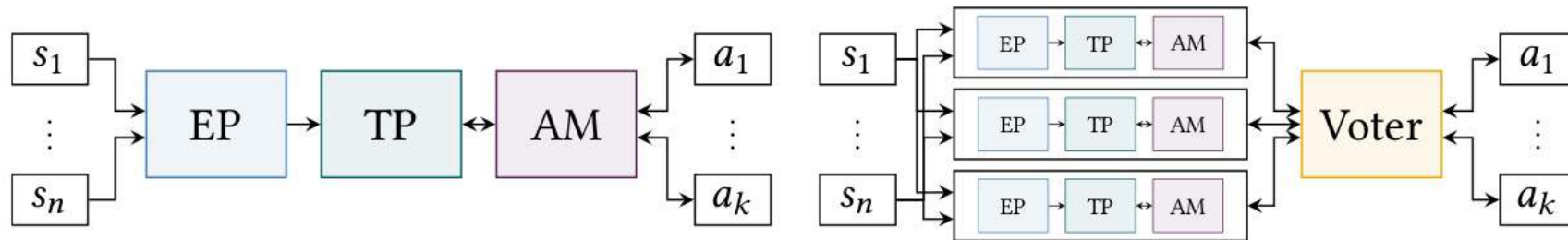


Major safety goal: **avoid wrong vehicle guidance.**

Automotive Safety Integrity **Level D**, i.e.,  $10^{-8}$  residual hardware failures per hour

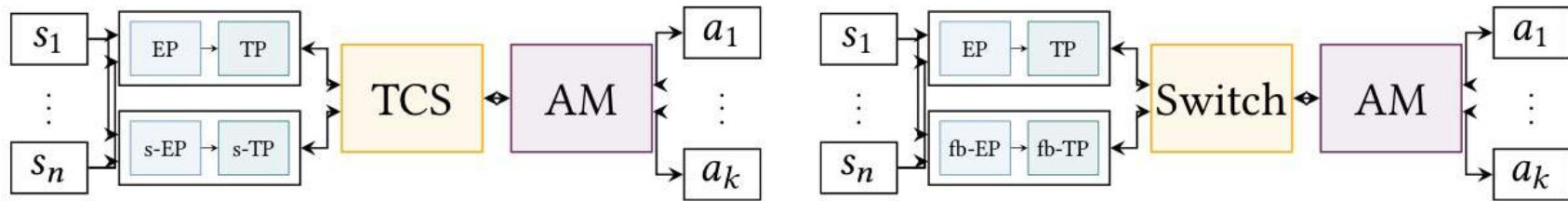


# Functional Safety Blocks



(a) Nominal function

(b) Triple modular redundancy (TMR)



(c) Nominal path and safety path

(d) Main path and fallback path

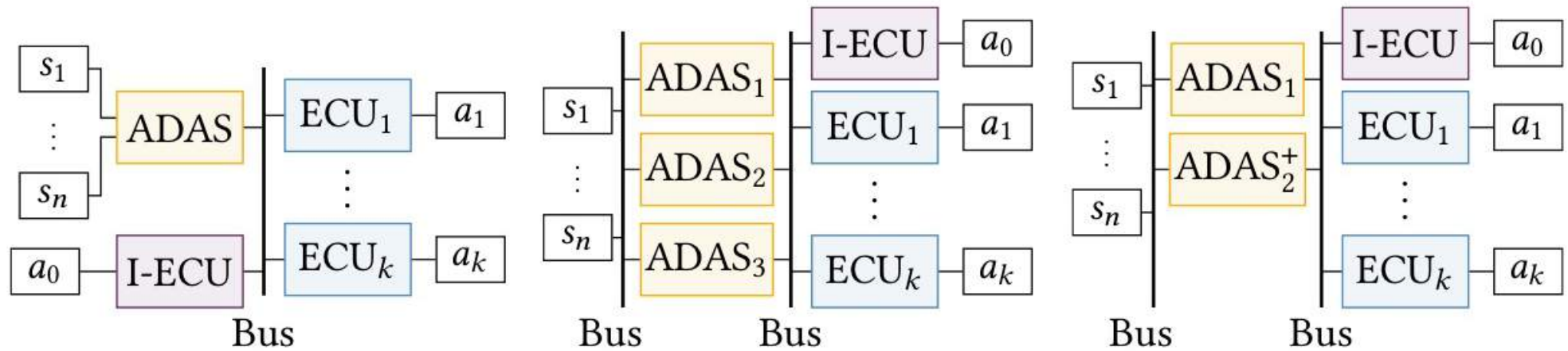
Fail-operational design patterns for autonomous driving.

EP = Environment Perception, TP = Trajectory Planning

AM = Actuator Mgt, TCS = Trajectory Checking and Selection



# Sample Car Architectures



(a) E/E architecture A

(b) E/E architecture B

(c) E/E architecture C

(a) **nominal**, (b) **“TMR”**, and (c) **ADAS+** architecture.

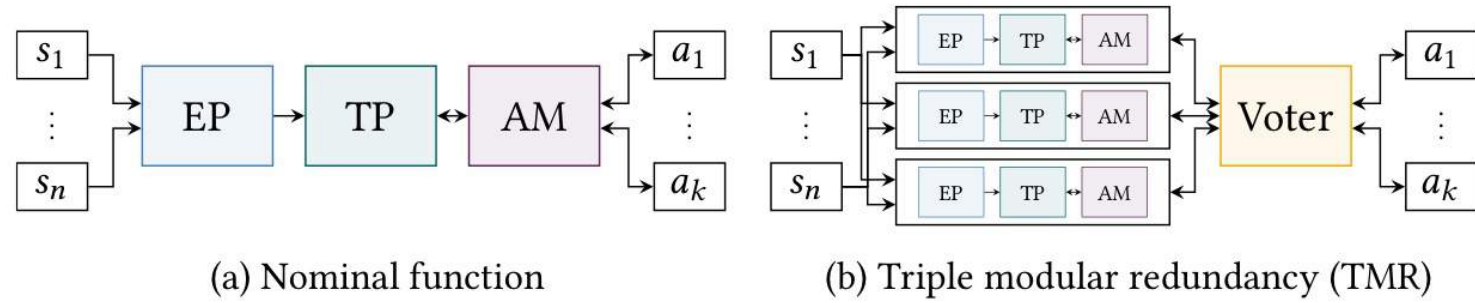
Assumption: during a transient fault, no other faults occur (conform ISO 26262)

ADAS = Advanced Driver Assistance System, I-ECU = Integration ECU

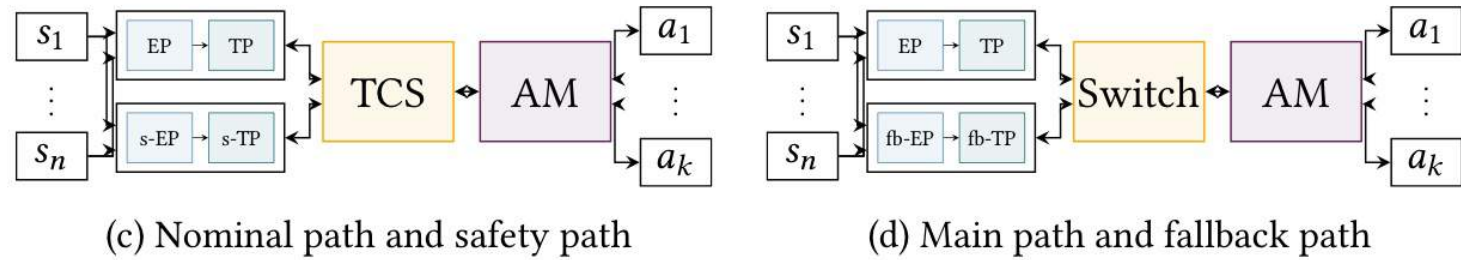




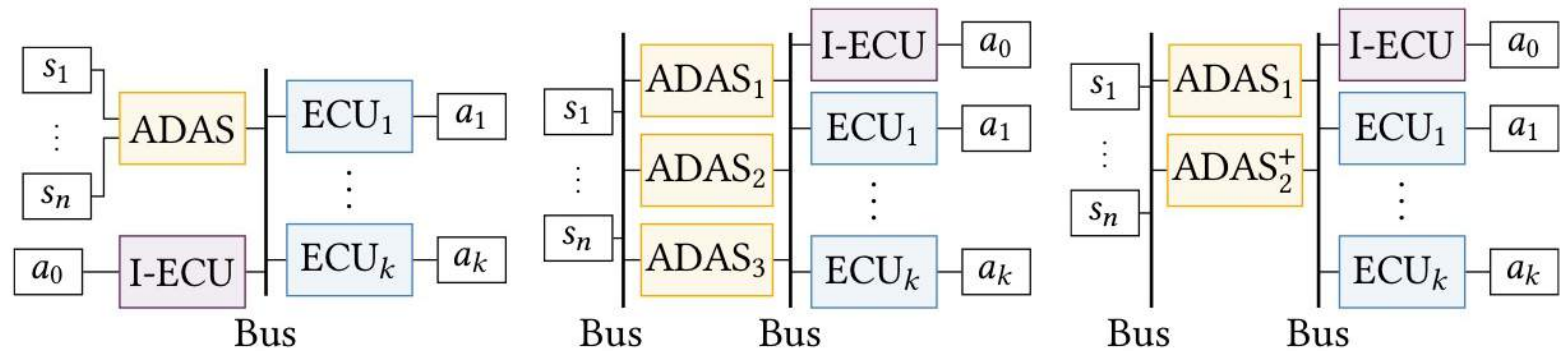
# Autonomous Vehicle Guidance



Software



Hardware





# Reliability Metrics Beyond Reliability and MTTF

---

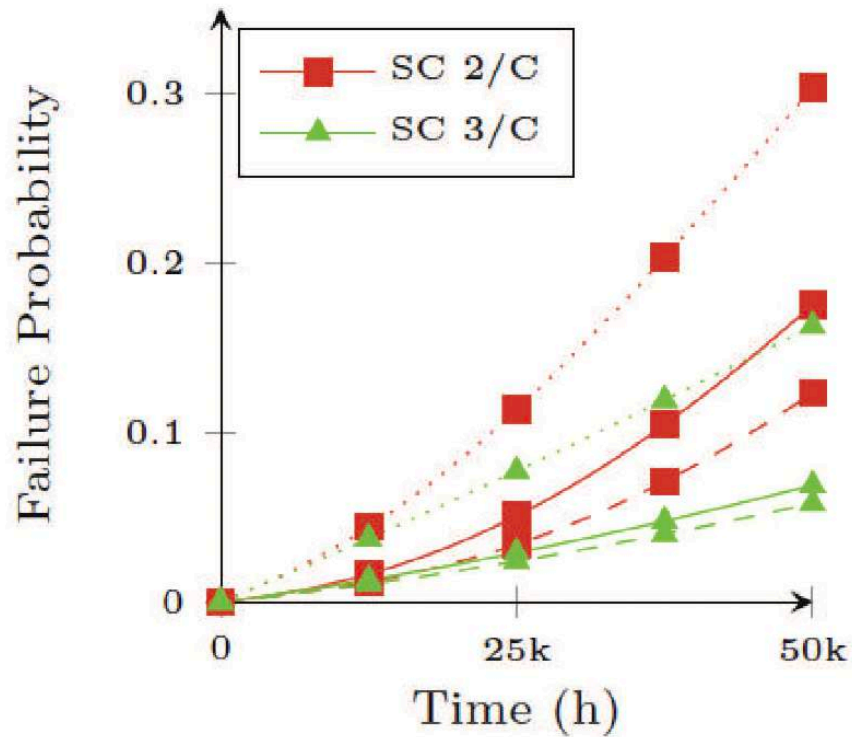
System integrity  $\approx$  probability of safe operation during operational lifetime

1. How probable is it that the system is fully functional at time  $t$ ?
2. What is the fraction of system failures w/o being degraded first?
3. The expected time to failure upon becoming degraded?
4. Criticality: how likely is it to fail within a drive cycle once degraded?
5. System integrity when limiting operational time after degradation?

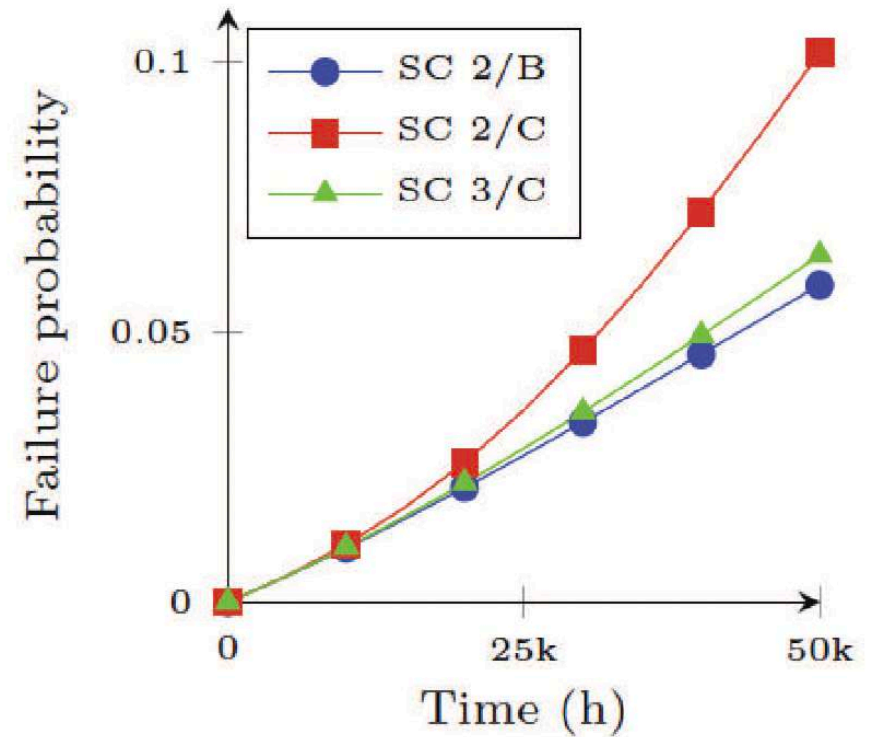


# DFT Modeling Statistics

	Scenario					DFT			CTMC		
	SC	Arch.	Adap.	Sens.	Act.	#BE	#Dyn.	#Elem.	#States	#Trans.	Degrad.
I	SC1	B	—	2/4	4/4	76	25	233	5,377	42,753	—
II	SC2	B	—	2/4	4/4	70	23	211	5,953	50,049	19.35%
III	SC2	C	ADAS+	2/4	4/4	57	19	168	1,153	7,681	16.65%
IV	SC3	C	—	2/4	4/4	57	21	170	385	1,985	12.47%
V	SC2	A	—	2/4	4/4	58	19	185	193	897	0.00%
VI	SC2	B	w/o I-ECU	2/4	4/4	65	21	199	1,201	8,241	19.98%
VII	SC2	B	5 ADAS	2/8	7/7	96	30	266	$2 \cdot 10^5$	$2 \cdot 10^6$	19.35%
VIII	SC2	B	8 ADAS	6/8	7/7	114	36	305	$4 \cdot 10^6$	$66 \cdot 10^6$	10.90%



Sensitivity



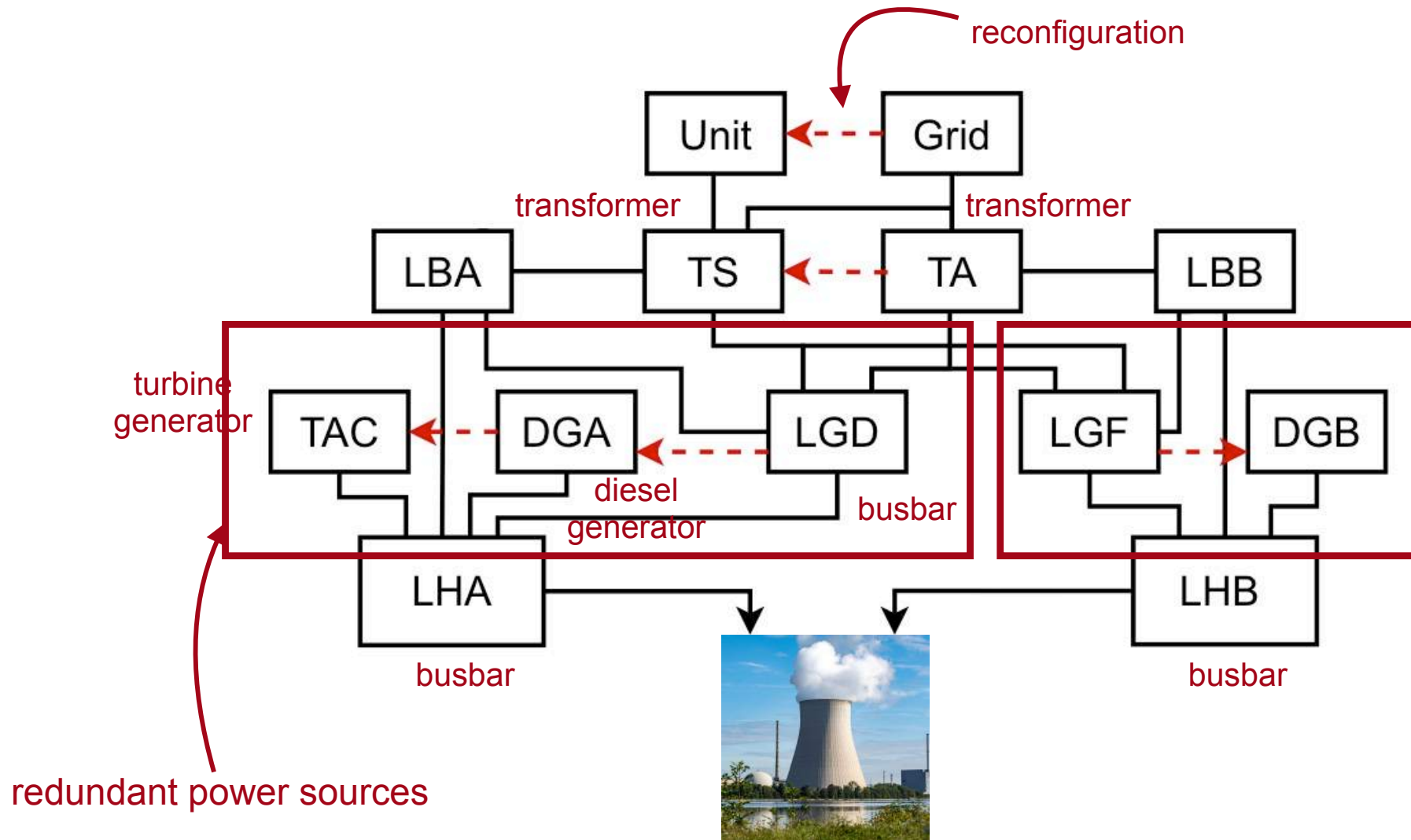
System integrity  
after degradation

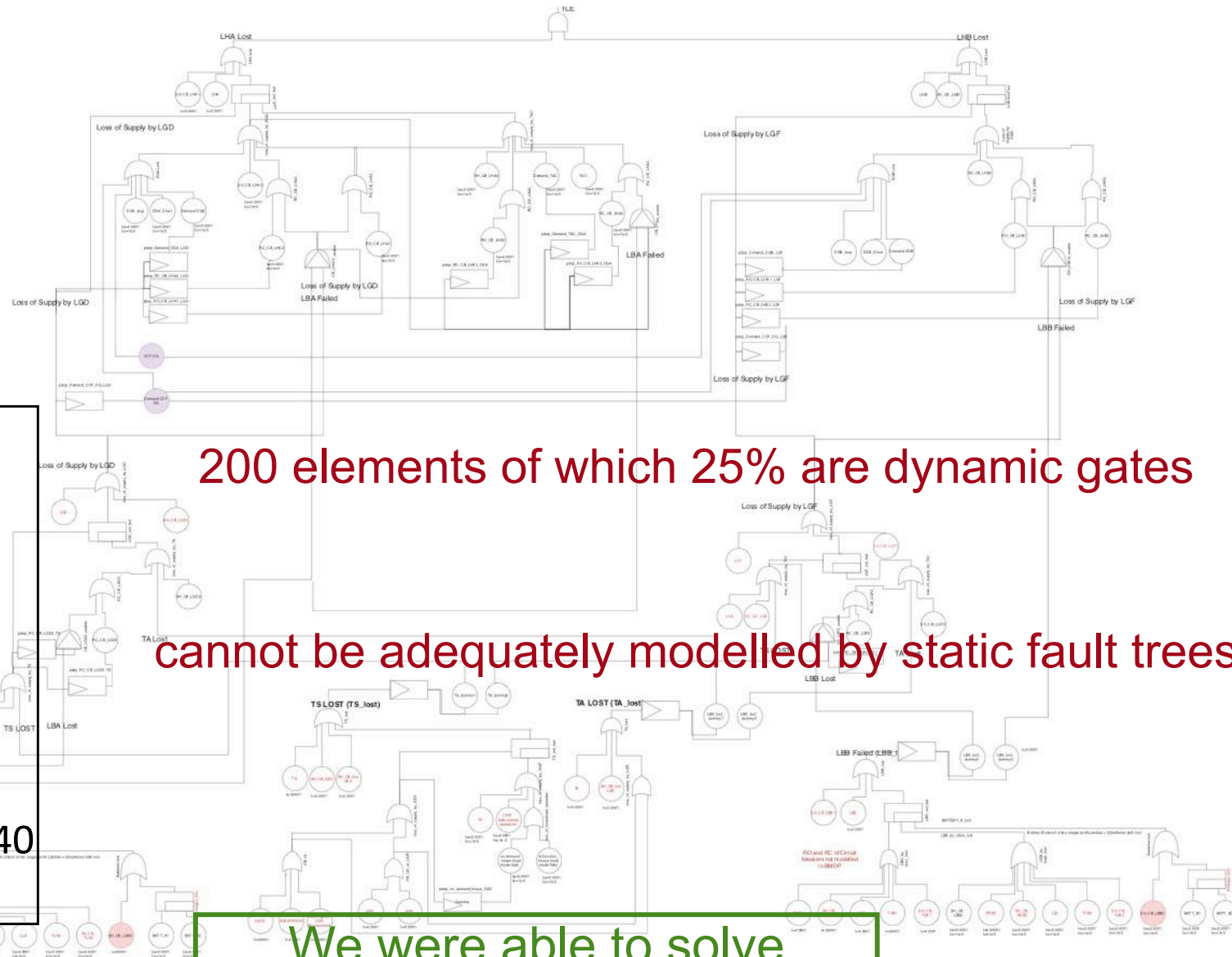
# Nuclear Power Plant

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- Nuclear Reactor managed by EDF – largest energy provider in France
- EDF challenged world reliability community to:
  - Faithfully model “Emergency Power Supply” and verify metrics like reliability, MTTF, etc.
- It is a highly complex and safety-critical system
  - Multiple power sources (high redundancy)
  - Large difference between failure rates of components
  - Components may fail:
    - Due to common cause failures (CCF)
    - While providing some functionality, e.g., generators fail while operating
    - When they are demanded for some service (on-demand failure)
  - Circular dependencies of components
  - Multi-directional interactions of components





BEs: 107

Static gates:

- AND: 2
- OR: 36

Dynamic gates:

- PAND: 5
- SPARE: 8
- PDEP/FDEP: 40
- SEQ: 2

200 elements of which 25% are dynamic gates

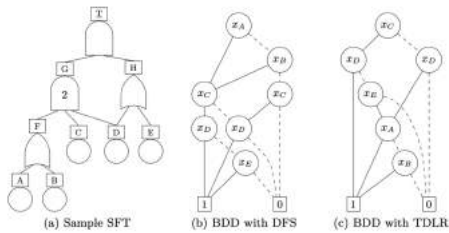
cannot be adequately modelled by static fault trees

We were able to solve this industrial challenge in 14s



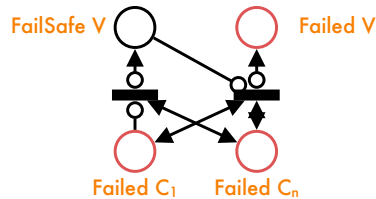
# Talk Overview

1.



## Classical Static Fault Trees

3.



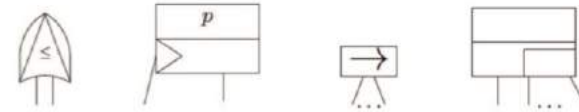
## Petri Net Semantics

5.



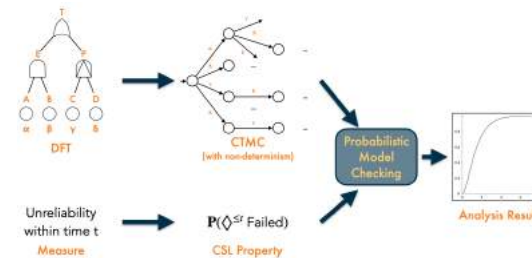
## Industrial Case Studies

2.



## Dynamic Fault Trees

4.



## Scaling Up DFT Analysis

6.



## Commercialisation

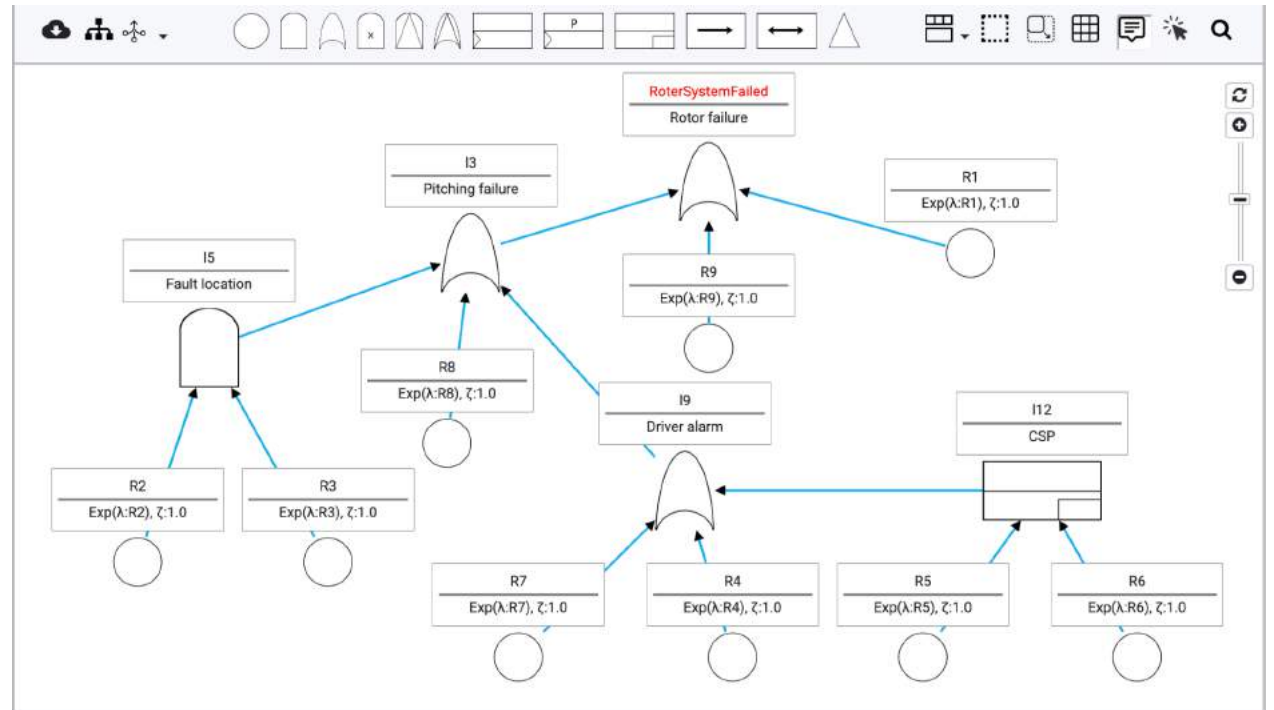
# SAFEST: Static And dynamic Fault trEe analySis Tool

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<https://www.safest.dgbtek.com>

# Modelling with SAFEST



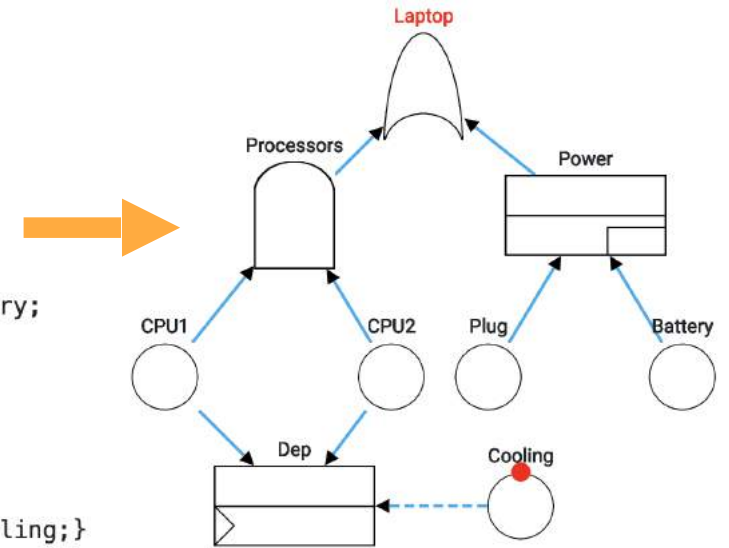
## Modelling

### 01 Graphical fault tree

# Modelling with SAFEST



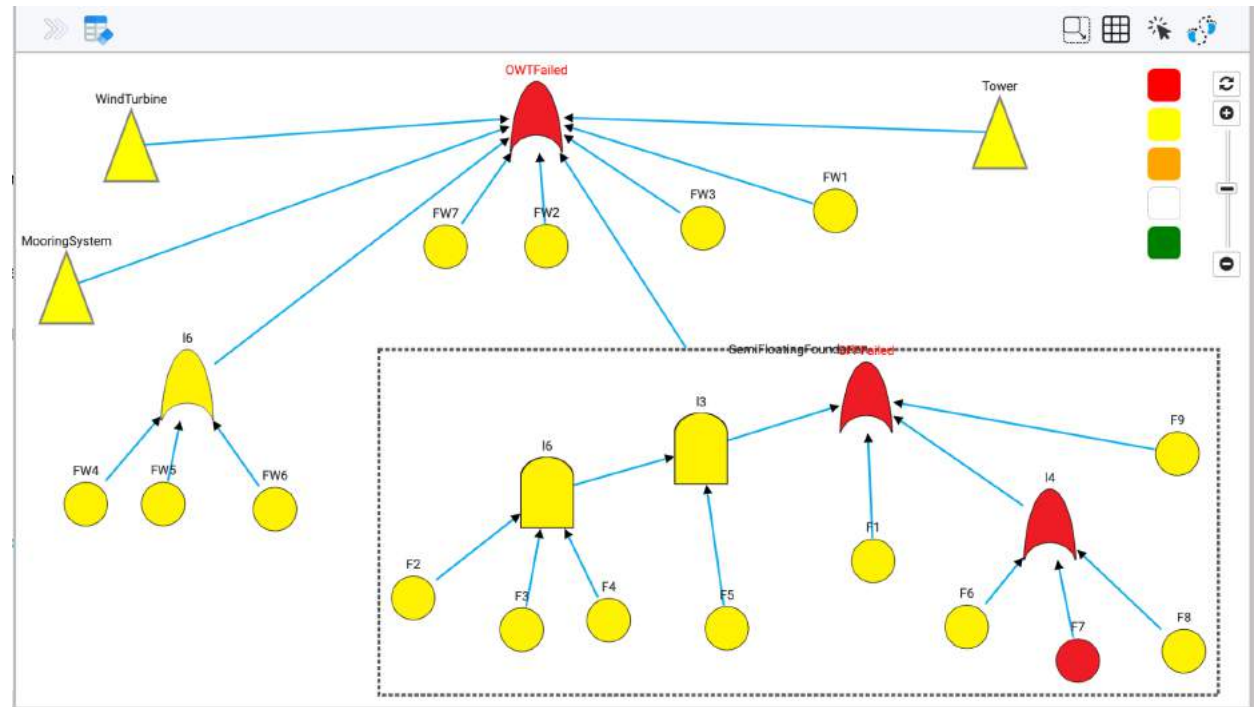
```
package LaptopPackage {  
  part Laptop{  
    part CPU1{↔}  
    part CPU2{↔}  
    part cooling{↔}  
    part plug{↔}  
    part battery{↔}  
  }  
  metadata power:REDUNDANCY about  
  Laptop::plug,noSwitch, Laptop::battery;  
  metadata processors:AND about  
  Laptop::CPU1, Laptop::CPU2;  
  metadata laptop:OR about  
  power, processors;  
  metadata Dep:FDEP about  
  Laptop::CPU1, Laptop::CPU2{  
    trigger_occurrence = Laptop::cooling;}  
  metadata TLE:TOP_LEVEL about laptop;  
}
```



## Modelling

02 SysML 2.0 to DFT

# Modelling with SAFEST

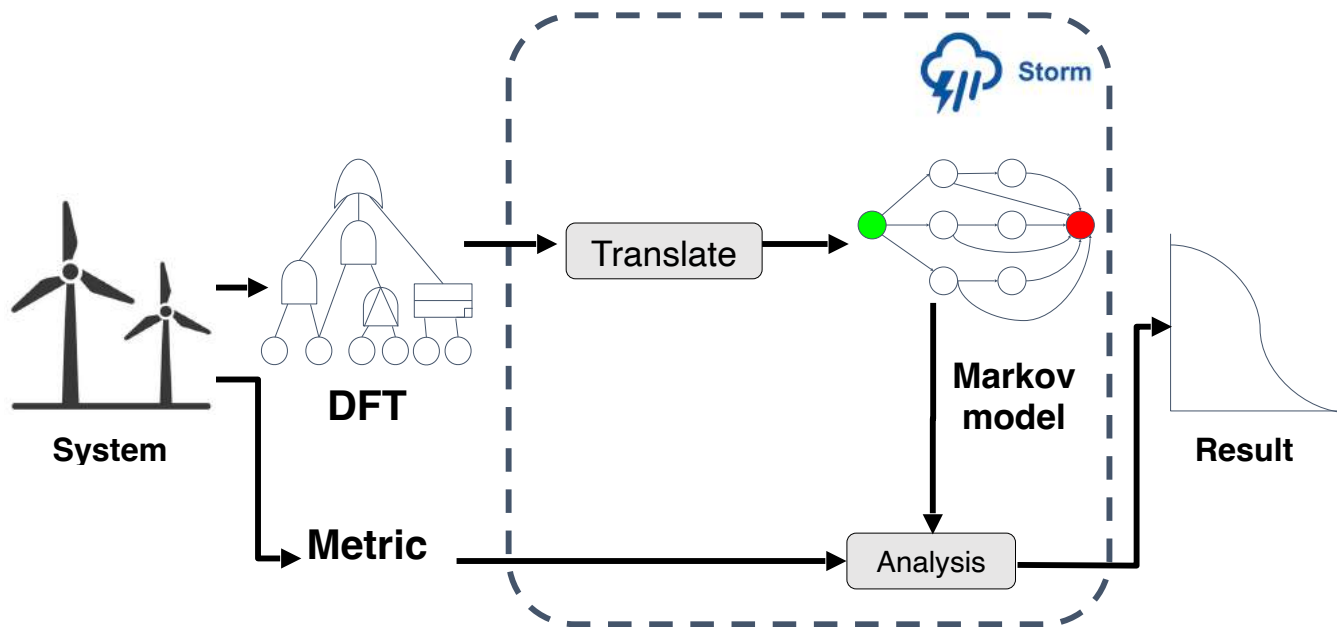


## Modelling

03

Interactive Simulation

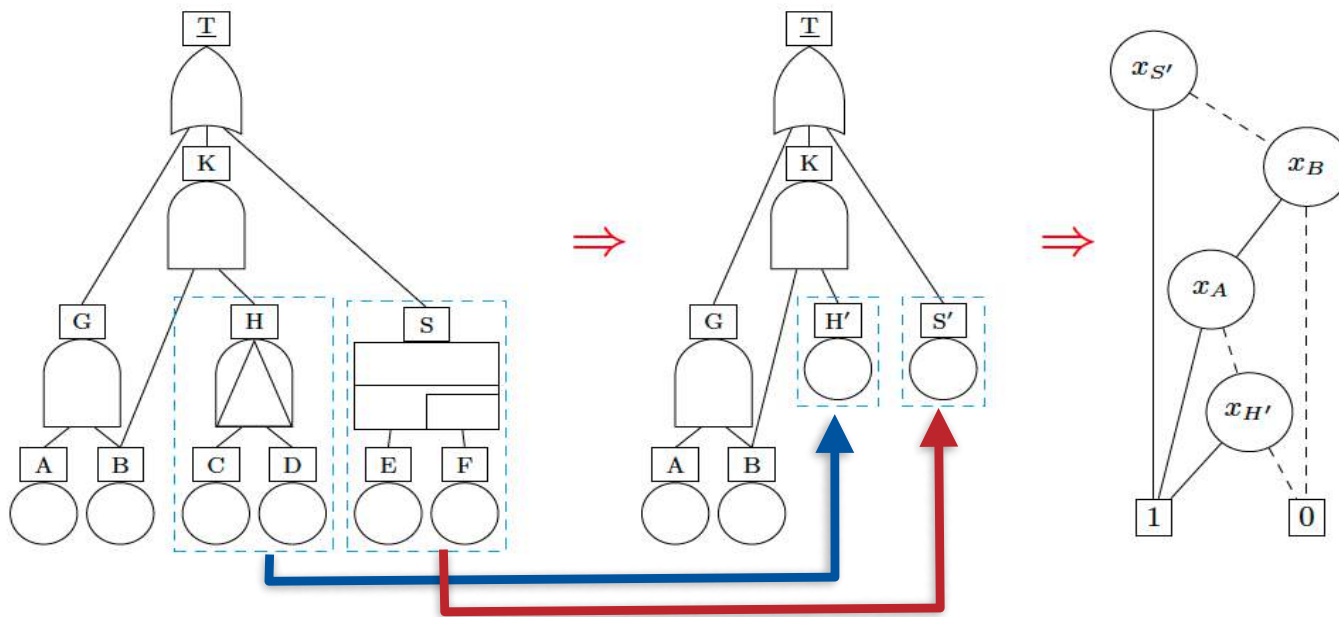
# Analysis with SAFEST



04 Probabilistic model checking

## Analysis

# Analysis with SAFEST

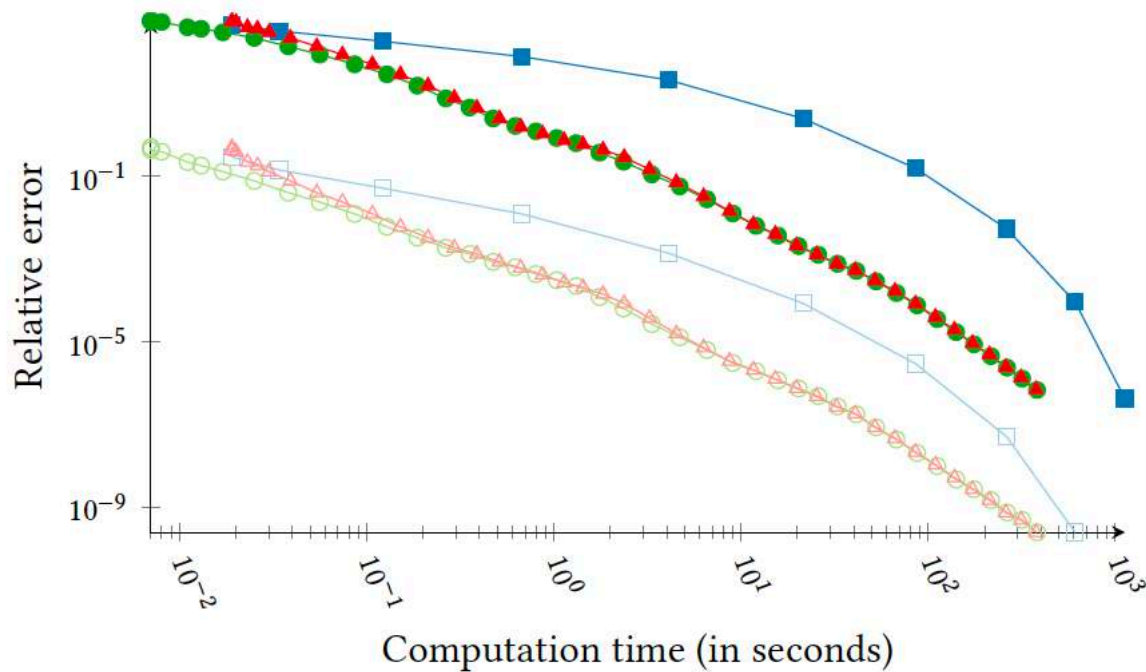


05 Modularisation



## Analysis

# Analysis with SAFEST



**06** Bounded approximation

## Analysis



# Take-Home Messages

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## What?

- Analysis of the largest dynamic fault trees ever
- Metrics beyond standard reliability measures
- Full automation: Storm-DFT --> SAFEST
- Validated by various industrial case studies

## How?

- DFT rewriting +
- Slim state-space generation +
- Tailored Markov chain model checking

## Try it out



We applied this principle also to BDMPs, an EDF fault tree dialect

## Literature

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- Semantic Intricacies of DFTs [Junges et al, [DSN 2016](#)]
- Simplifying DFTs by Graph Rewriting [Junges et al, [Form. Asp. Comp., 2017](#)]
- Fast DFT Analysis by Model Checking [Volk et al, [IEEE Trans. Ind. Inf, 2018](#)]
- One Net Fits All: Unifying Semantics of DFTs [Volk et al, [Petri Nets 2018](#)]
- Analysing DFTs with Static Parts [Basgöze et al, [NFM 2022](#)]
- Railway Station Areas Application [Weik et al, [STTT 2022/FMICS 2019](#)]
- Autonomous Car Application [Ghadhab et al, [Reliab. Eng. Syst. Saf. 2019](#)]
- Reliability Analysis of EDF's Fault Trees [Khan et al, [DSN 2021/IEEE TDSC 2023](#)]
- SAFEST: Static and Dynamic Fault Tree Aanalysis Tool [Volk et al., [ESREL 2023](#)]

# Big Thanks to my Co-Workers!

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Matthias Volk



Sebastian Junges



Marielle Stoelinga

Arend Rensink

Falak Sher

Nils Nießen

Mattias Kuntz

Shahid Kahn

Enno Ruijters

Norman Weik

Dennis Guck

Daniel Basgöze

Madji Ghadhab

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