PLANNING AND GAMES

ENCODING IN SAT AND QBF

… AND QUANTUM CIRCUIT COMPILATION

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PLAN

1. **Motivating example: Quantum Circuit Layout Synthesis**

2. Classical Planning / Bounded Model Checking (SAT)

3. Concise Encoding of Planning in QBF (Quantified Boolean Formulas)
    - Path Compression
    - Lifted Planning (almost first-order)

4. Concise Encoding of 2-player board games in QBF
    - Tic-tac-toe, Hex, Breakthrough, Domineering

5. Validation of encodings with QBF certificates

INTRODUCTION – LAYOUT SYNTHESIS

- *Optimal Layout Synthesis for Quantum Circuits as Classical Planning,*
  Irfansha Shaik and Jaco van de Pol. ICCAD'23 IEEE/ACM, San Francisco, 2023

Quantum algorithms can solve some problems faster than classical algorithms

Quantum Computers exist today … NISQ era

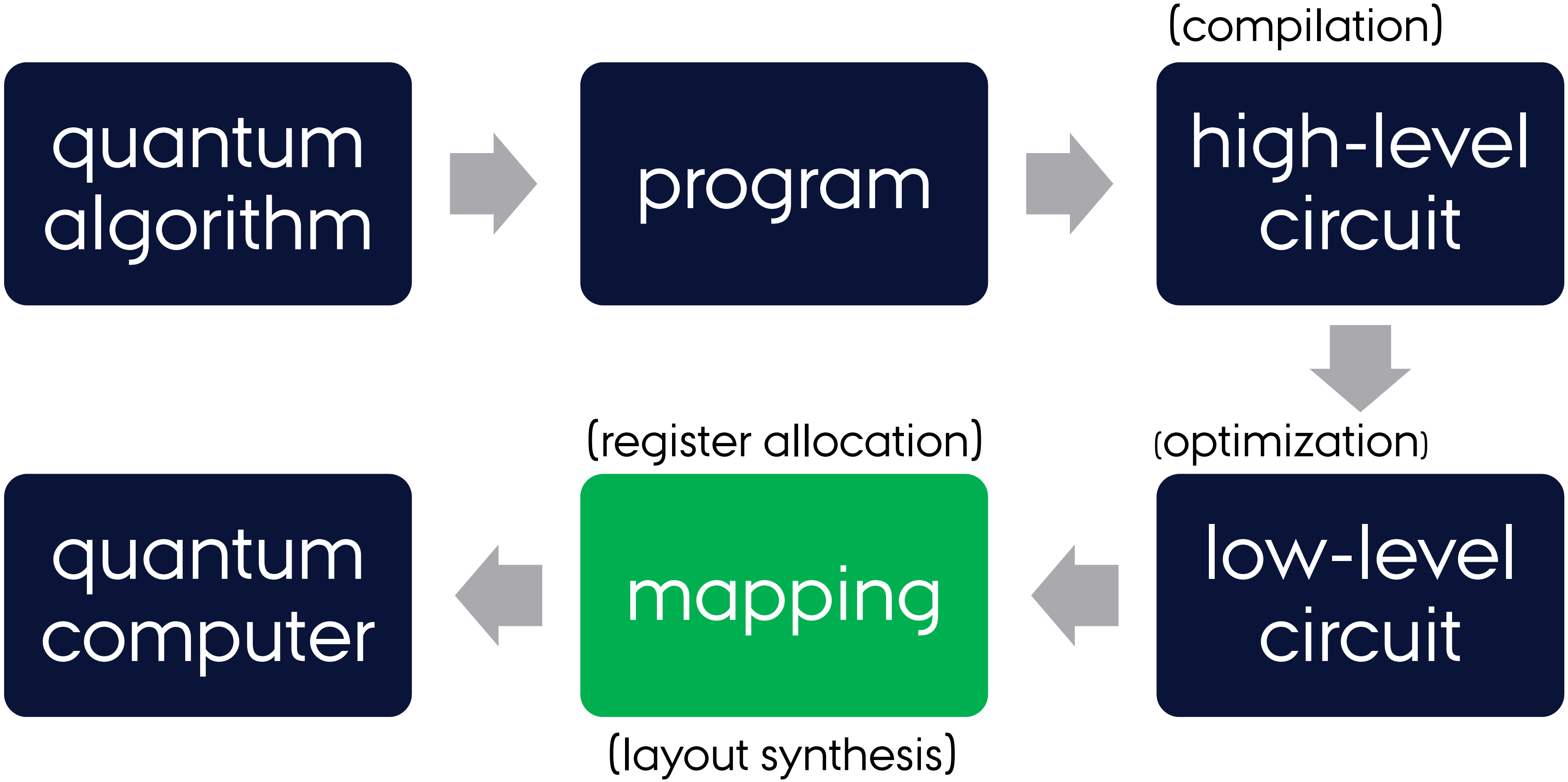
- Intermediate scale (limited number of qubits, limited connectivity)
- Noisy (decoherence, interference)

Circuit Optimization: minimize the number of gates / depth of circuit by rewriting the circuit

Layout Synthesis: Map "logical quantum circuit" to a "physical platform"

- Swap qubits around, to obey connectivity restrictions
- Every swap increases the noise, so minimize this!

A SMALL STEP IN THE "QUANTUM PIPELINE"



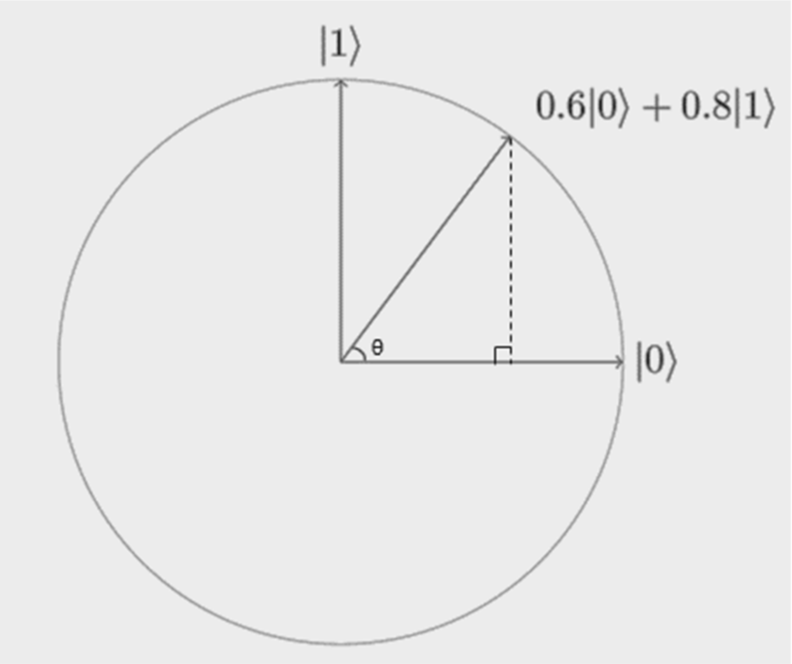
quantum algorithm → program → (compilation) high-level circuit → (optimization) low-level circuit → (register allocation) mapping (layout synthesis) → quantum computer

QUBITS AND QUANTUM GATES (HEAVILY SIMPLIFIED)

Qubits:

Basic vectors: $|0\rangle = \binom{1}{0}$ and $|1\rangle = \binom{0}{1}$

Arbitrary qubit: $|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$ with $|\alpha|^2 + |\beta|^2 = 1$



1 qubit Quantum Gates: $I, X, Y, Z, H, S, T, T^\dagger$, …

Identity = I = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\quad I(x) = x$

NOT = X = $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ $\quad NOT(x) = \neg x$

MULTI-QUBIT QUANTUM GATES (CLASSICAL VIEW)

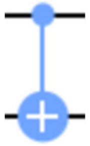
Multiple qubits: $|\phi\psi\rangle = |\phi\rangle \otimes |\psi\rangle$ (tensor product)

$|00\rangle = \begin{pmatrix}1\\0\\0\\0\end{pmatrix}$, $|01\rangle = \begin{pmatrix}0\\1\\0\\0\end{pmatrix}$, $|10\rangle = \begin{pmatrix}0\\0\\1\\0\end{pmatrix}$ and $|11\rangle = \begin{pmatrix}0\\0\\0\\1\end{pmatrix}$

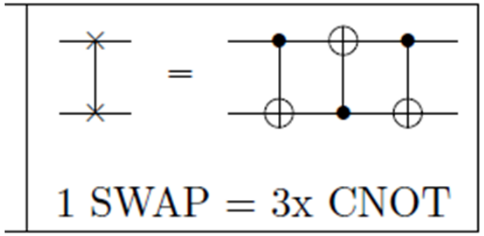
n qubits: 2^n-dimensional Hilbert space

Binary operators:

$$\text{CNOT} = \text{CX} = \begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 0 & 1\\0 & 0 & 1 & 0\end{bmatrix}$$



$$CNOT(x,y) = (x, x \oplus y)$$



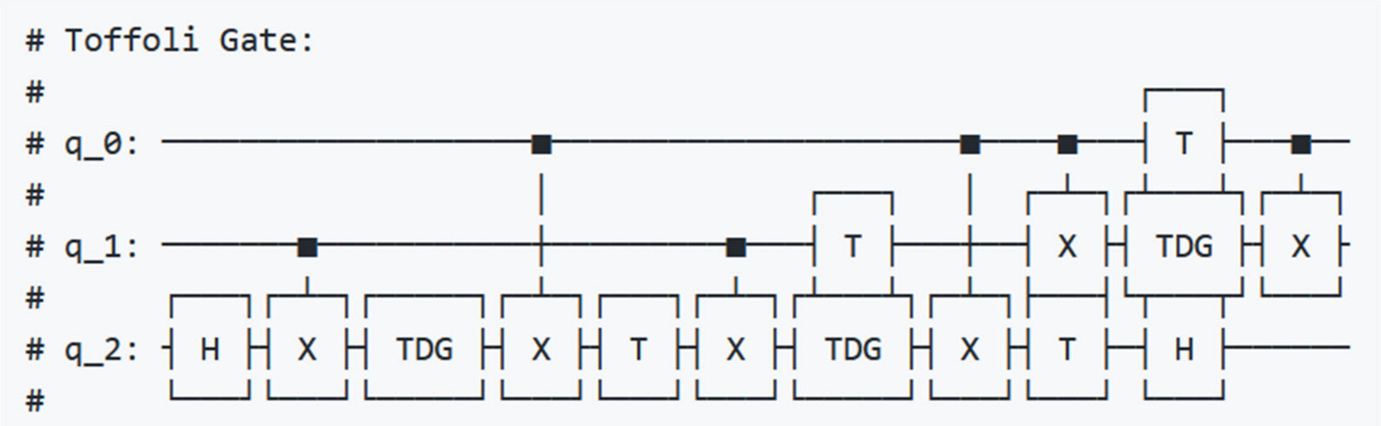
1 SWAP = 3x CNOT

$$\text{SWAP} = \begin{bmatrix}1 & 0 & 0 & 0\\0 & 0 & 1 & 0\\0 & 1 & 0 & 0\\0 & 0 & 0 & 1\end{bmatrix}$$

$$\text{SWAP}(x,y) = (y,x)$$

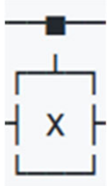
LAYOUT SYNTHESIS

(FROM GITHUB.COM/UCLA-VAST/OLSQ)



Toffoli Gate:

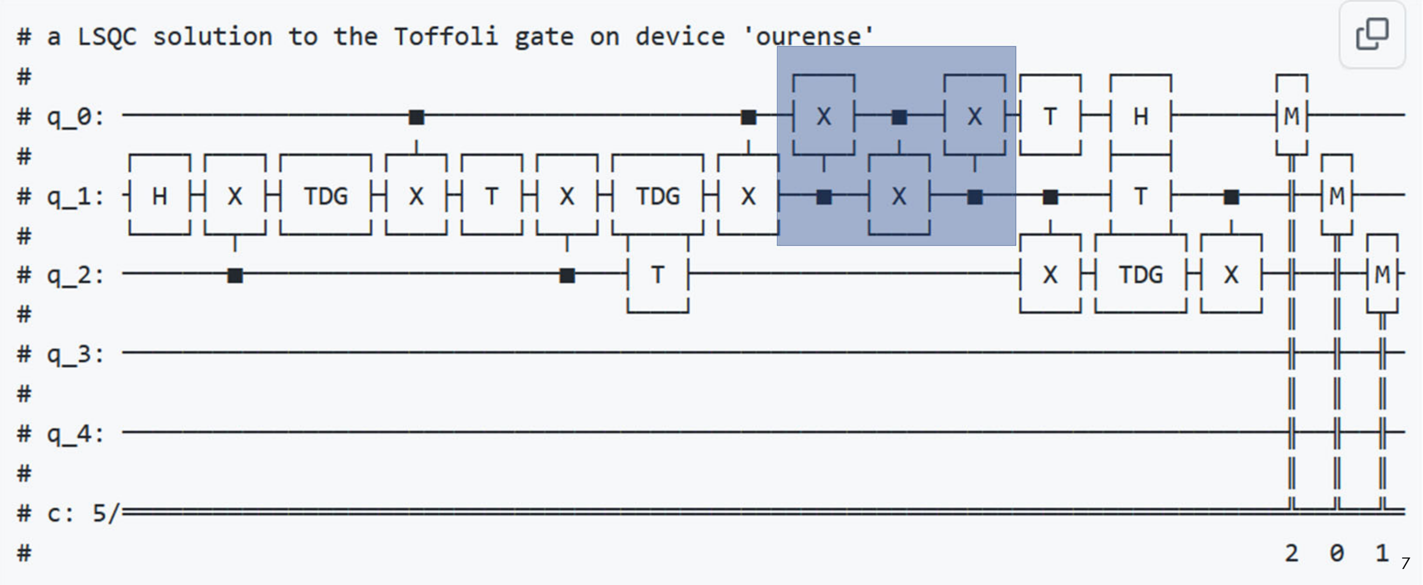
All gates (H,X,T) are unary except binary CNOT gate:



Requires full connectivity $(q_0, q_1), (q_1, q_2), (q_0, q_2)$

What if the target only has $(q_0, q_1), (q_1, q_2)$?

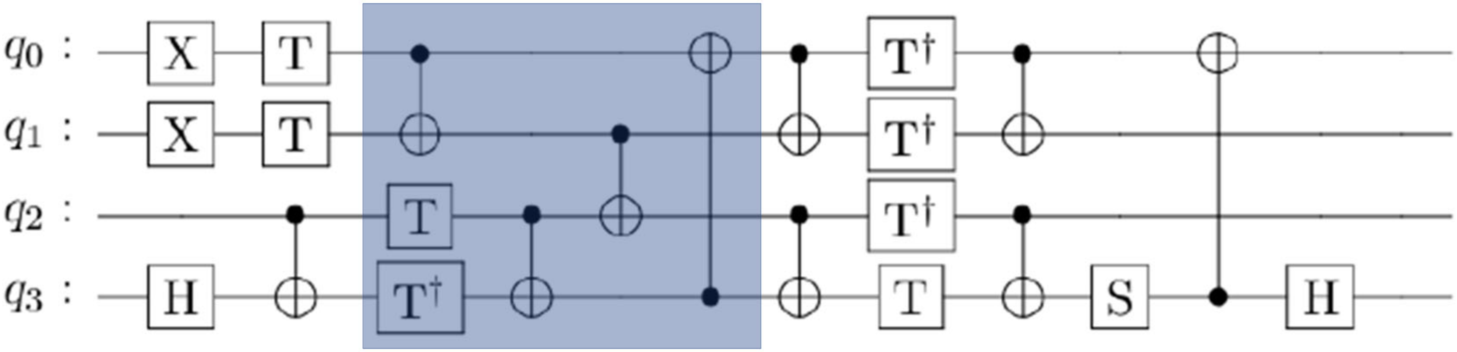
1 SWAP by 3 CNOT gates



a LSQC solution to the Toffoli gate on device 'ourense'

LAYOUT SYNTHESIS: ADDER CIRCUIT

First input: a (logical) quantum circuit

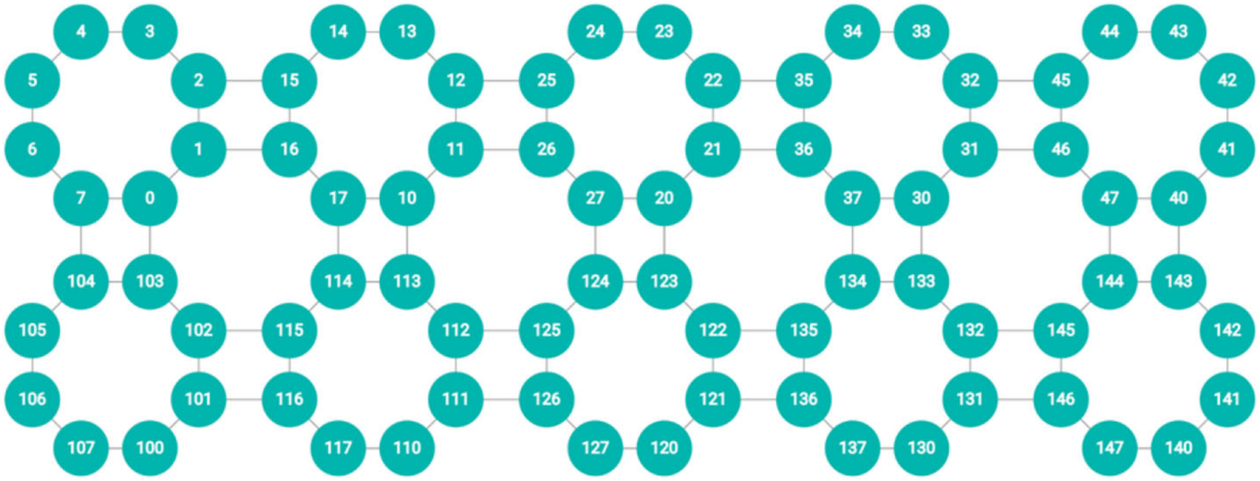


Note: Need a square: $(q_0, q_1), (q_1, q_2), (q_2, q_3), (q_3, q_1)$

What if the physical platform only has $(q_0, q_1), (q_1, q_3), (q_2, q_3)$?

PHYSICAL PLATFORMS

Second input: a (directed) Coupling Map



Rigetti Aspen-3, 80 qubits (source: aws.amazon.com)

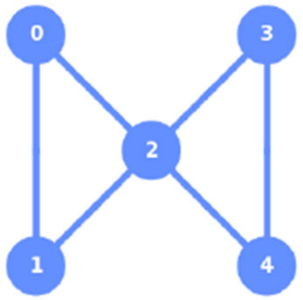


Fig. 3. Coupling map for IBM-QX2 (Tenerife)

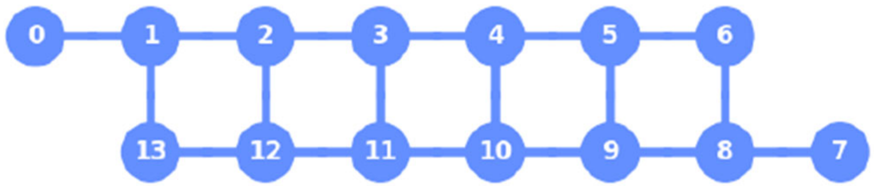
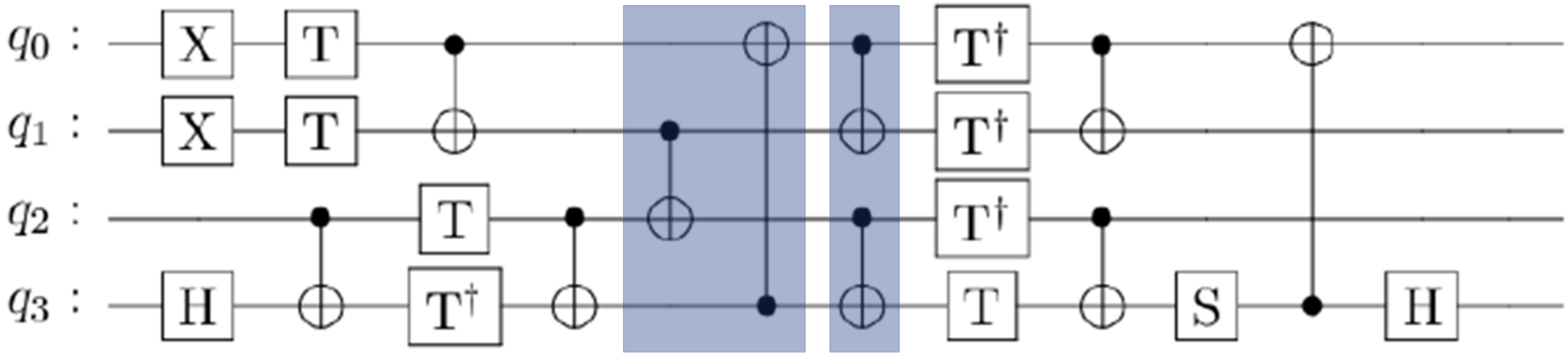
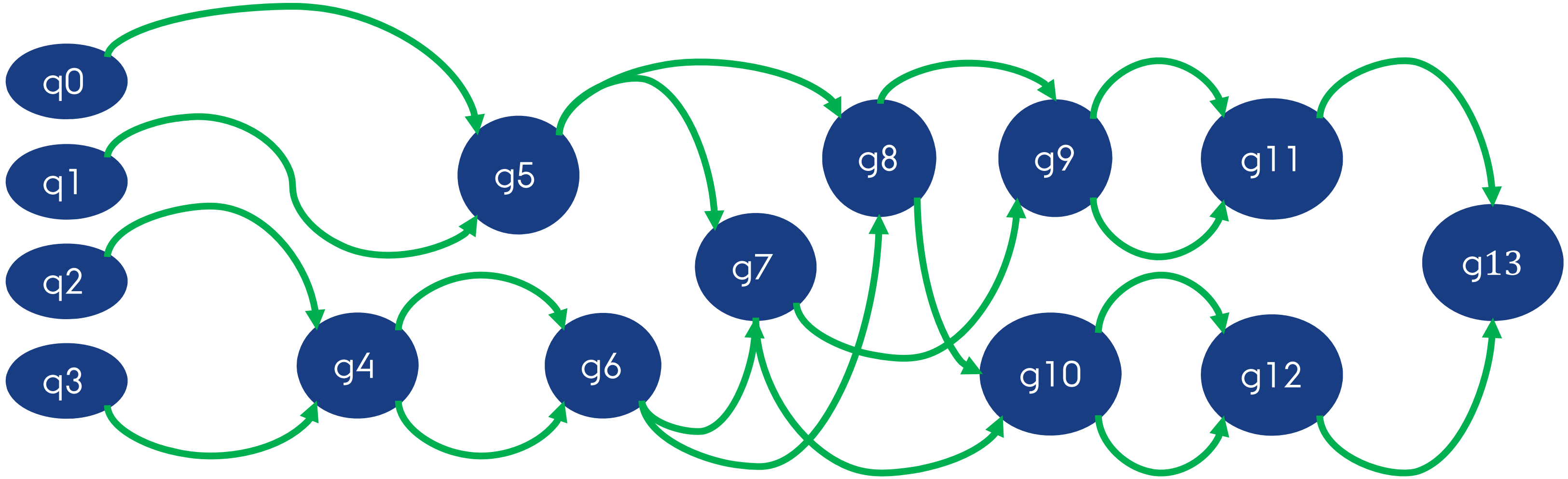


Fig. 4. Coupling map for IBM Melbourne

DEPENDENCY GRAPH (DAG)

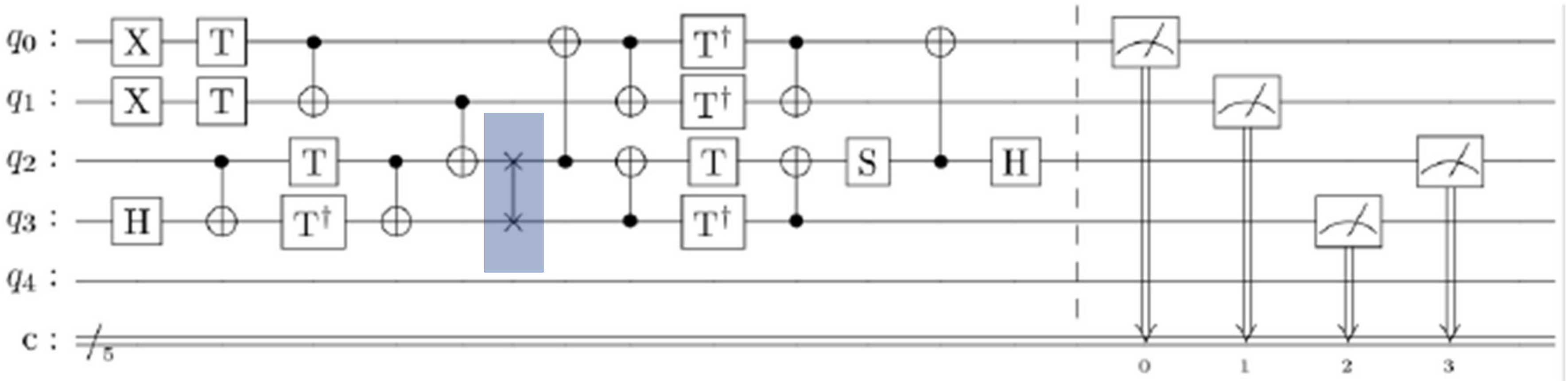


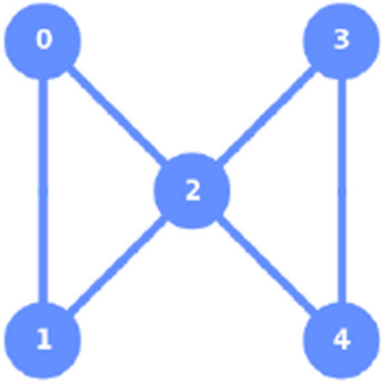
For layout mapping, we can focus on the CNOT gates and ignore all unary gates



ADDER – ON 5-QUBIT PLATFORM

Result: a new quantum circuit, observing "neighbours", inserting minimal #SWAP gates





Apparently, the optimal solution for this example uses only 1 SWAP

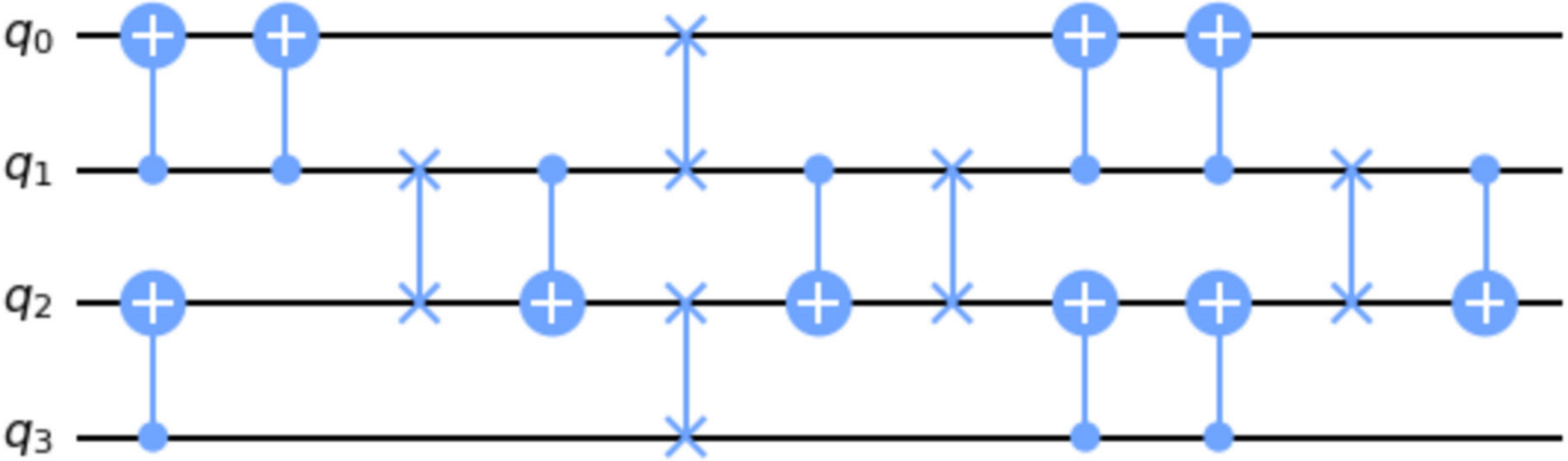
Note: after the swap, the "logical qubits" are on different "physical qubits"

(indicated by the "measurements")

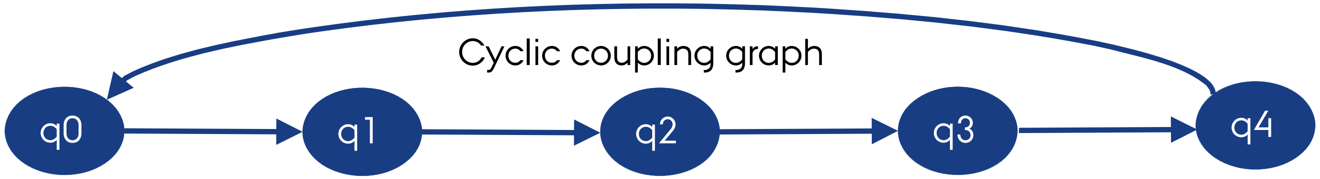
Complicated factor: in general, one may need extra "ancillary" qubits! (q4)

MORE CHALLENGING – RESTRICT PLATFORM

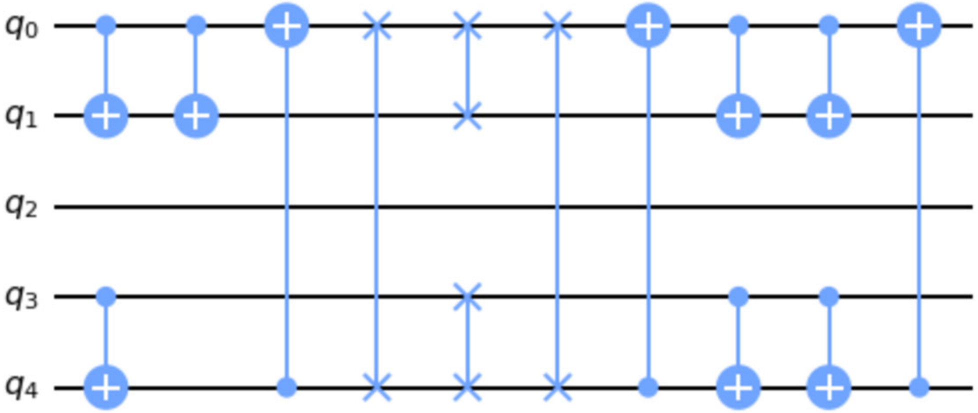
Optimal number of swaps: 5



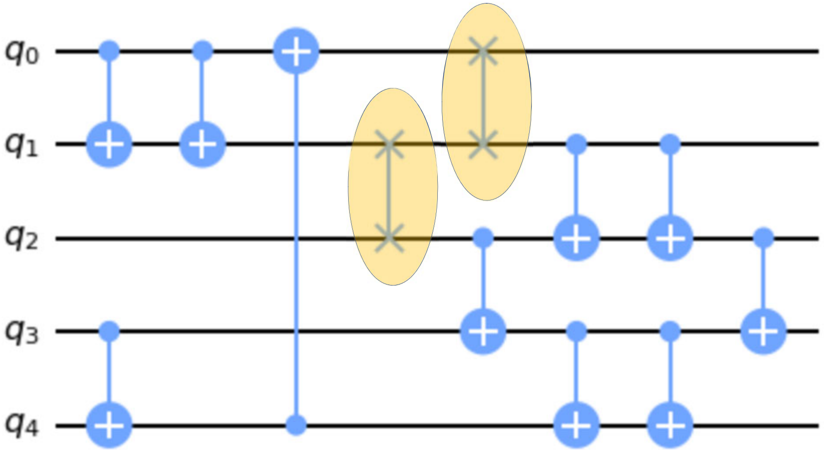
NEED FOR ANCILLARY QUBITS



Without using ancillary bits (4 swaps)



With using ancillary bits (2 swaps)



LESSONS INSPIRED BY CONCURRENCY

The notion of equivalence determines "optimality"

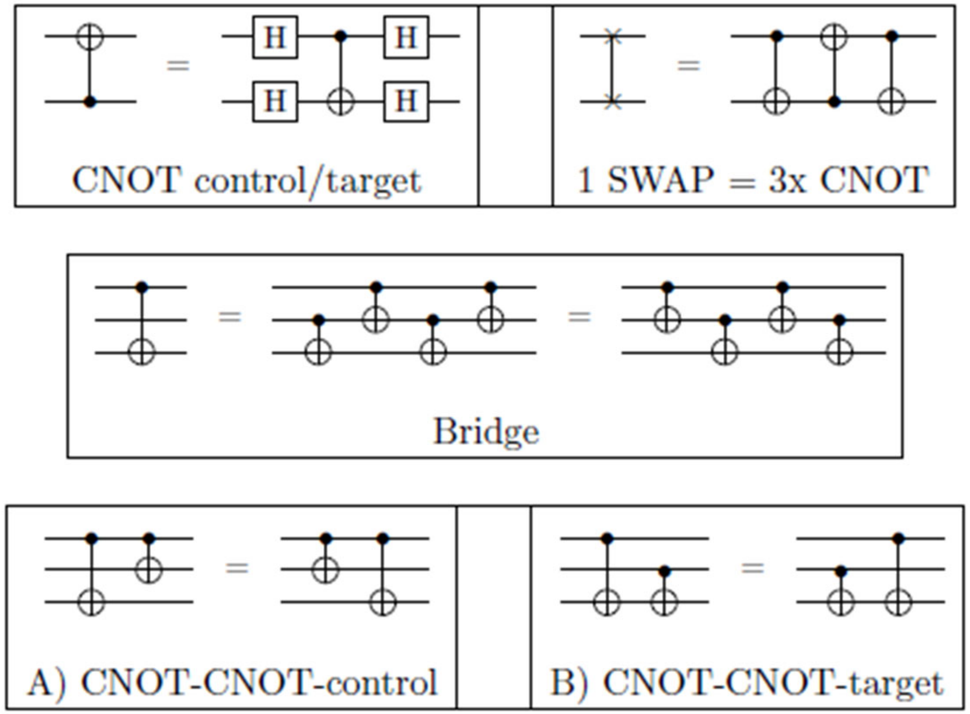
Should we preserve

- the same list of gates?
- only "local" dependency DAG? (POR)
- "layers" in a topological sorting? (parallelism)

Adder example		
On cycle-platform	With ancillary	2 swaps
	Without ancillary	4 swaps
On star-platform	Total topological order	8 swaps
	Partial order preserved	6 swaps

Can we take advantage of "semantic" equalities?

Where is the limit? Complete re-synthesis?



[Itoko et al., in: Integration, 2020]

PLAN

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CLASSICAL PLANNING (PDDL)

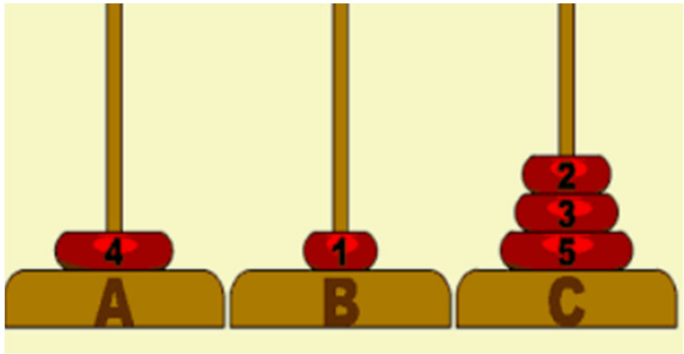
Domain description:

States:

- Described by predicates
- Can be tested and updated

Actions: (parametric)

- Described by pre-conditions and effects
- Both are conjunctions of predicates

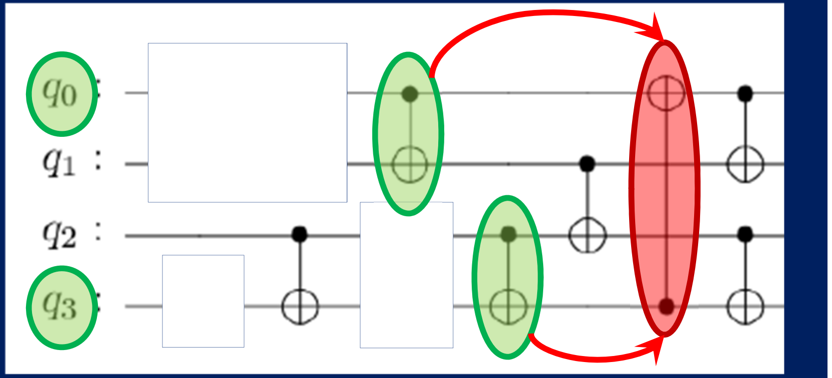


Problem instance:

- Concrete objects
- Initial state, Goal state(s)

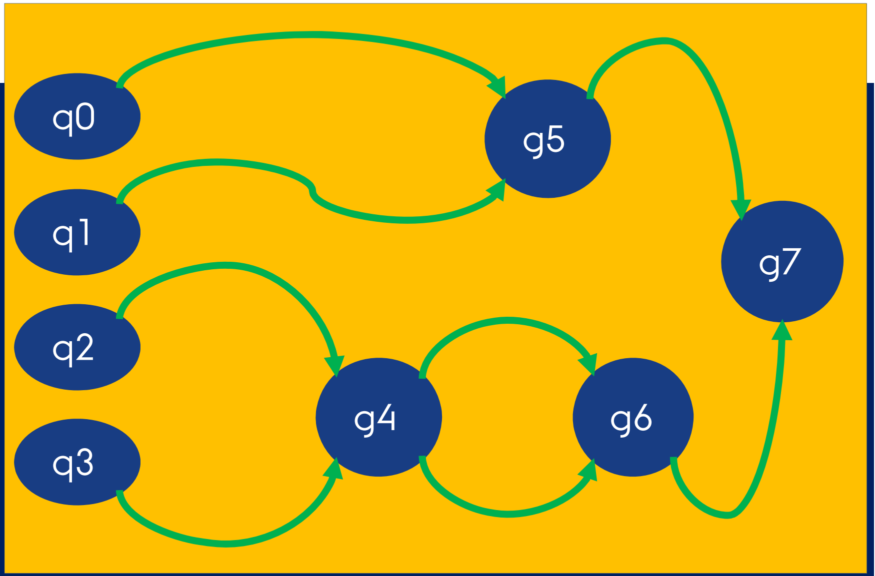
LAYOUT SYNTHESIS → PLANNING

(OUTSTANDING DOMAIN SUBMISSION AWARD IPC-2023)



```
(define (domain quantum)
(:types
    gate - object   ; binary or input gate
    pbit - object   ; physical qubit
    lbit - gate     ; logical qubit
)
(:predicates
     (mapped ?l - lbit  ?p - pbit)                   ; l is mapped on p
     (used ?p - pbit)                                ; p is in use
     (done ?g - gate)                                ; gate g is done
     (neighbour ?p1 ?p2 - pbit)                      ; static connections
     (cnot ?l1 ?l2 - lbit  ?g0 ?g1 ?g2 - gate)      ; gate dependencies
) … )
```

LAYOUT SYNTHESIS: PROBLEM FILE



```
(define (problem adder) (:domain quantum)
(:objects
    l0 l1 l2 l3 - lbit
    p0 p1 p2 p3 p4 - pbit
    g4 g5 g6 g7 g8 g9 g10 g11 g12 g13 - gate
)
(:init ; static predicates: platform & circuit
    (neighbour p0 p1) (neighbour p1 p0) (neighbour p3 p4) ...
    (cnot l2 l3 g4 l2 l3) (cnot l0 l1 g5 l0 l1) (cnot l1 l2 g7 g5 g6) ...
)
(:goal
    (and (done g13))
))
```

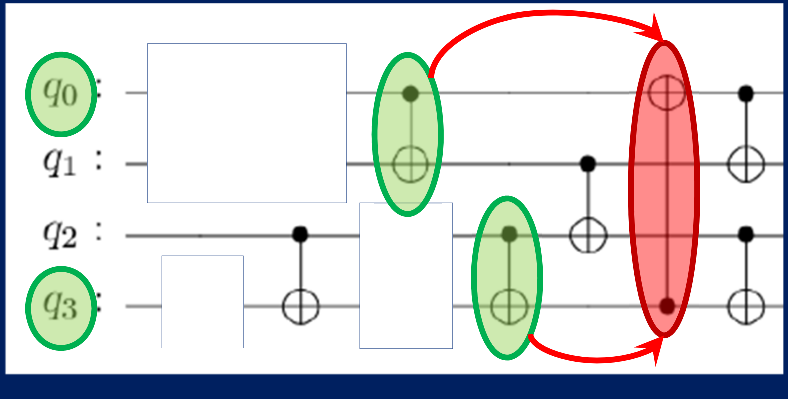
ACTIONS: MAP INITIAL + SWAP

```
(:action map_init
    :parameters
        (?l1 - lbit ?p1 - pbit)
    :precondition (and
        (not (used ?p1))
        (not (done ?l1))
    )
    :effect (and
        (mapped ?l1 ?p1)
        (used ?p1)
        (done ?l1)
    )
)
```

```
(:action swap
    :parameters
        (?l1 ?l2 - lbit ?p1 ?p2 - pbit)
    :precondition (and
        (neighbour ?p1 ?p2)
        (mapped ?l1 ?p1)
        (mapped ?l2 ?p2)
    )
    :effect (and
        (not (mapped ?l1 ?p1))
        (not (mapped ?l2 ?p2))
        (mapped ?l1 ?p2)
        (mapped ?l2 ?p1)
    )
)
```

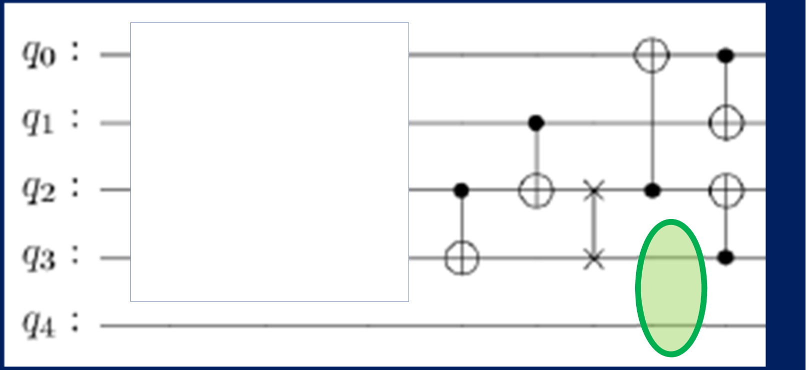
ACTIONS: APPLY A CNOT GATE

```
(:action apply_cnot
    :parameters
        (?l1 ?l2 - lbit ?p1 ?p2 - pbit ?g0 ?g1 ?g2 - gate)
    :precondition (and
        (cnot ?l1 ?l2 ?g0 ?g1 ?g2)
        (neighbour ?p1 ?p2)
        (mapped ?l1 ?p1) (mapped ?l2 ?p2)
        (done ?g1) (done ?g2) (not (done ?g0))
    )
    :effect (and
        (done ?g0)
    )
)
```



COMPLETENESS: USE ANCILLARY QUBIT

```
(:action use_ancillary
    :parameters
        (?l1 - lbit ?p1 ?p2 - pbit)
    :precondition (and
        (neighbour ?p1 ?p2)
        (mapped ?l1 ?p1)
        (not (used ?p2))
    )
    :effect (and
        (not (mapped ?l1 ?p1)) (not (used ?p1))
        (mapped ?l1 ?p2) (used ?p2)
)   )
```



we also need the symmetric one with (neighbor ?p2 ?p1)

EFFICIENCY: COMBINE INIT_MAP IN CNOT

```
(:action apply_cnot_input_input
    :parameters (?l1 ?l2 - lbit ?p1 ?p2 - pbit ?g0 - gate)
    :precondition (and
        (cnot ?l1 ?l2 ?g0 ?l1 ?l2)
        (neighbour ?p1 ?p2)
        (not (used ?p1)) (not (used ?p2))
        (not (done ?g0)) (not (done ?l1)) (not (done ?l2))
    )
    :effect (and
        (done ?g0) (done ?l1) (done ?l2)
        (mapped ?l1 ?p1) (used ?p1)
        (mapped ?l2 ?p2) (used ?p2)
)   )
```

we also need two variants with one input and one gate

EFFICIENCY 2: GROUNDING ALL GATES

```
(:action apply_cnot_g7        ; g7 depends on CNOT gates g5, g6
    :parameters (?p1 ?p2 - pbit)
    :precondition (and
        (neighbor ?p1 ?p2)
        (mapped l1 ?p1) (mapped l2 ?p2)
        (done g5) (done g6) (not (done g7)))
    :effect (and (done g7)))
(:action apply_cnot_g4        ; g4 depends on input gates l2, l3
    :parameters?p1 ?p2 - pbit)
    :precondition (and (neighbor ?p1 ?p2)
       (not (used ?p1)) (not (used ?p2)) (not (done g4)))
    :effect (and (done g4)
        (mapped l2 ?p1) (used ?p1)
        (mapped l3 ?p2) (used ?p2)))
```

IMPLEMENTATION: Q-SYNTH

Our tool Q-SYNTH

Implemented in Python:

- Input: quantum circuit + coupling graph → Output: PDDL planning instance
- Uses QISKIT for I/O and computing the dependency graph

We need a "planning tool", running in "optimal mode"

→ take winners from IPC – international planning competitions

- Madagascar – SAT based (similar to BMC)
- Fast Downward Soup

Shortest Plan → quantum circuit: re-insert the unary gates

COMPARING TO RELATED WORK

SABRE (QISKIT): Tackling the qubit mapping problem for NISQ-era quantum devices
G. Li, Y. Ding, and Y. Xie (ASPLOS 2019)

- Heuristic, fast, but not-optimal

QMAP: Mapping Quantum Circuits to IBM QX Architectures
Robert Wille, Lukas Burgholzer and Alwin Zulehner (DAC-2019)

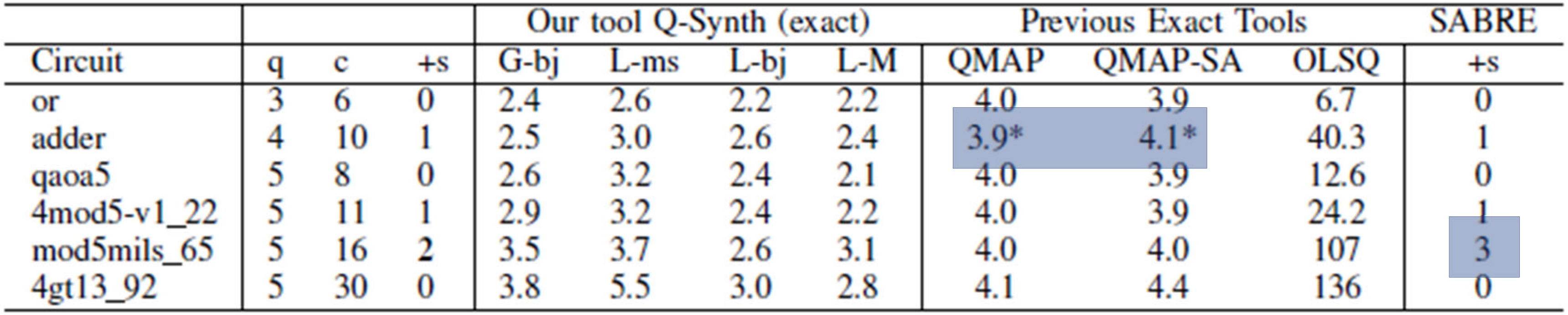
- Used **SAT + SMT** technology
- Considers all permutations of qubits

OLSQ: Optimal Layout Synthesis for Quantum Computing,
Bochen Tan and Jason Cong (ICCAP 2020)

- Also **SAT + SMT**, exponentially less variables, avoids permutations

SMALL PLATFORM – 5 QUBITS

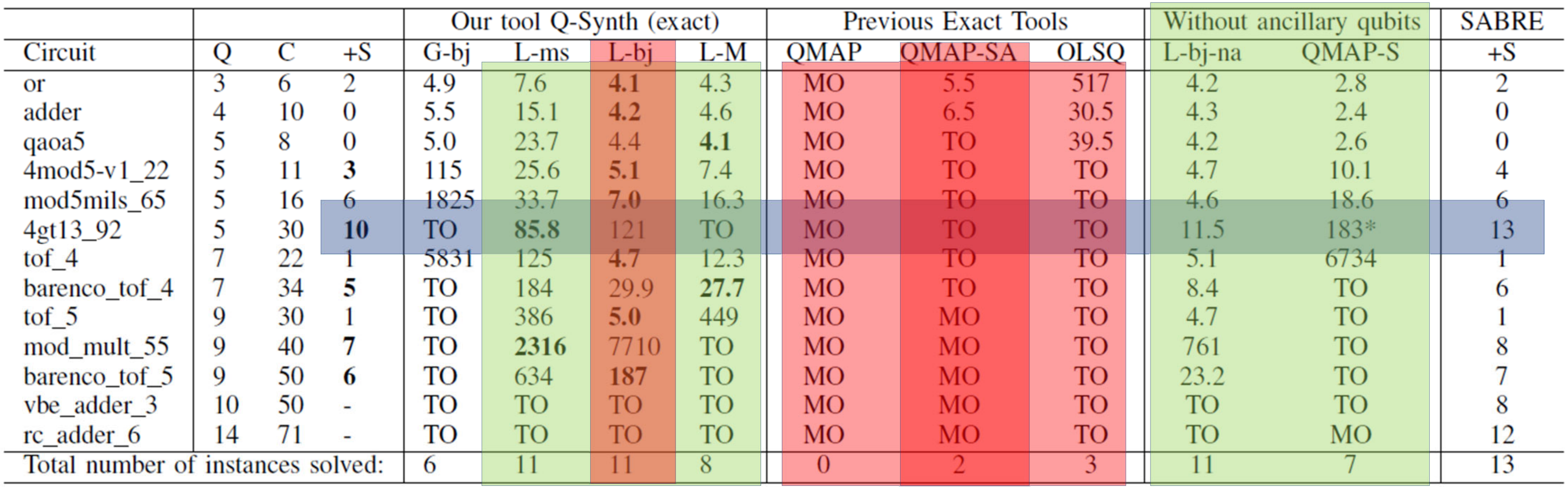
(TIME IN SEC)



				Our tool Q-Synth (exact)				Previous Exact Tools			SABRE
Circuit	q	c	+s	G-bj	L-ms	L-bj	L-M	QMAP	QMAP-SA	OLSQ	+s
or	3	6	0	2.4	2.6	2.2	2.2	4.0	3.9	6.7	0
adder	4	10	1	2.5	3.0	2.6	2.4	3.9*	4.1*	40.3	1
qaoa5	5	8	0	2.6	3.2	2.4	2.1	4.0	3.9	12.6	0
4mod5-v1_22	5	11	1	2.9	3.2	2.4	2.2	4.0	3.9	24.2	1
mod5mils_65	5	16	2	3.5	3.7	2.6	3.1	4.0	4.0	107	3
4gt13_92	5	30	0	3.8	5.5	3.0	2.8	4.1	4.4	136	0

- Note: all tools work, Q-synth is fast and always optimal
  - G-bj: Global Encoding (lifted, separate init)
  - L-* : Local Encoding (grounded, integrated init)
- SABRE is non-optimal (by design)
- QMAP is non-optimal in a few cases (total order)
- OLSQ might also be non-optimal (fixed horizon)

LARGER PLATFORM – 14 QUBITS (3HR TIMEOUT, 48 GB MEM)



				Our tool Q-Synth (exact)				Previous Exact Tools			Without ancillary qubits		SABRE
Circuit	Q	C	+S	G-bj	L-ms	L-bj	L-M	QMAP	QMAP-SA	OLSQ	L-bj-na	QMAP-S	+S
or	3	6	2	4.9	7.6	**4.1**	4.3	MO	5.5	517	4.2	2.8	2
adder	4	10	0	5.5	15.1	**4.2**	4.6	MO	6.5	30.5	4.3	2.4	0
qaoa5	5	8	0	5.0	23.7	4.4	**4.1**	MO	TO	39.5	4.2	2.6	0
4mod5-v1_22	5	11	**3**	115	25.6	**5.1**	7.4	MO	TO	TO	4.7	10.1	4
mod5mils_65	5	16	6	1825	33.7	**7.0**	16.3	MO	TO	TO	4.6	18.6	6
4gt13_92	5	30	**10**	TO	**85.8**	121	TO	MO	TO	TO	11.5	183*	13
tof_4	7	22	1	5831	125	**4.7**	12.3	MO	TO	TO	5.1	6734	1
barenco_tof_4	7	34	**5**	TO	184	29.9	**27.7**	MO	TO	TO	8.4	TO	6
tof_5	9	30	1	TO	386	**5.0**	449	MO	MO	TO	4.7	TO	1
mod_mult_55	9	40	**7**	TO	**2316**	7710	TO	MO	MO	TO	761	TO	8
barenco_tof_5	9	50	**6**	TO	634	**187**	TO	MO	MO	TO	23.2	TO	7
vbe_adder_3	10	50	-	TO	TO	TO	TO	MO	MO	TO	TO	TO	8
rc_adder_6	14	71	-	TO	TO	TO	TO	MO	MO	TO	TO	MO	12
Total number of instances solved:				6	11	11	8	0	2	3	11	7	13

- SABRE is non-optimal: can use 13 Swaps instead of 10 = 9 extra CNOT gates (!)
- Q-SYNTH is the only exact tool that scales: *9 logical qubits on 14 physical qubits*
- The planner backends are somewhat complementary
- Planning without using ancillary bits is considerably easier

CONCLUSION, PERSPECTIVES

- Classical Planning for optimal layout synthesis gives superior results
- Classical Planning could also make use of heuristic planners (suboptimal plans)

Even better encodings are possible, to limit the search space

- The sub-architecture technique from QMAP can also be applied in Q-SYNTH
- Parallel plans, exploit symmetries, relaxed dependencies

- ***Current/Future work:*** *Direct SAT encoding, …*
- ***Future work:*** *Other costs (swap depth, noise reduction, …)*
- ***Future work:*** *Quantum Circuit Optimization seems a much harder problem*

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PLANNING: BOUNDED MODEL CHECKING

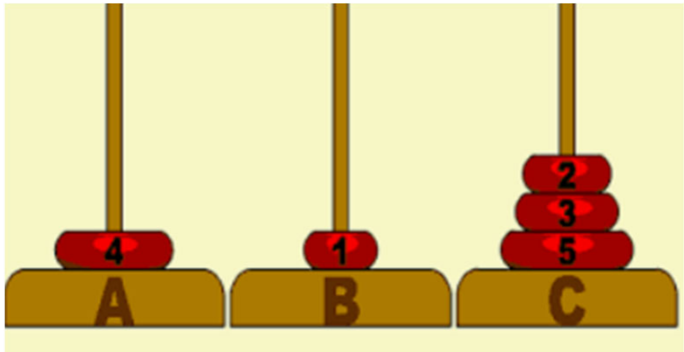
Domain description:

States (S): Described by predicates

Actions (A): Described by pre-conditions and effects

Problem instance:

- Concrete objects
- Initial state (I), Goal state (G)



Bounded Model Checking / Planning as Satisfiability [Kautz, Selman 1992]

- State variables S_i per time step
- Transition relation A captures actions
- Reduce to SAT solver (plan of 2 actions)

$$\exists S_0, S_1, S_2 : I(S_0) \land A(S_0, S_1) \land A(S_1, S_2) \land G(S_2)$$

LIMITATION OF BMC WITH SAT IN PLANNING

The encoding may become very large:

- The transition relation can become quite large
  - **Grounding**: Instantiate actions for all **object combinations**

$$\boldsymbol{apply\_cnot(l_1, l_2, p_1, p_2, g_1, g_2, g_3)}$$

  - **Copying:** The transition relation is copied for each step
- Existing solution (in planning and bounded model checking):
  - “Iterative squaring”, also known as “path compression”
  - Encode in QBF: only one copy of transition relation A

QBF AND CONCISE ENCODING

QBF extends Propositional Logic with quantifiers over propositions

Only one of the following formulas is true:

1. $\forall x\, \exists y : x \land y$
2. $\exists y\, \forall x : x \land y$
3. $\forall x\, \exists y : x \leftrightarrow y$
4. $\exists y\, \forall x : x \leftrightarrow y$

- Let ϕ be a very large formula: how can we encode $\phi(A) \land \phi(B)$ concisely?

$$\forall x : (x = A \lor x = B) \to \phi(x)$$

- Here we use only one copy of ϕ

QBF SOLVERS

SAT is a special case of QBF – only ∃-quantifiers – NP-hard

In general, solving QBF is PSPACE-complete

The **quantifier alternation depth** is a crucial factor for complexity

Practical QBF solvers exist, there is also a yearly QBF-EVAL competition

(we contributed our planning and game encodings to QBF-EVAL)

Example QBF solvers:

CAQE	[Rabe, Tentrup]
DepQBF	[Lonsing, Biere]
…	

We proposed a translator from PDDL to QBF (ICAPS 2022)

PLAN

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3. **Concise Encoding of Planning in QBF**
    - **Path Compression**
    - Lifted Planning (almost first-order)

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    - Tic-tac-toe, Hex, Breakthrough, Domineering

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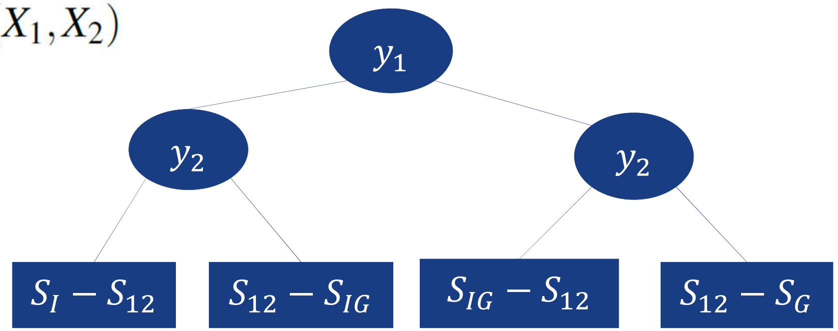
FLAT AND COMPACT TREE ENCODING

Flat Encoding

- Only one copy of transition relation
- $\log(k) + 2$ quantifier alternations

[Rintanen, 2003]
[Dershowitz etal, 2005]
[Jussila, Biere, 2007]

$$\begin{aligned}
&\exists S_I, S_G\\
&\exists S_{IG} \forall y_1 \exists S_1, S_2\\
&\exists S_{12} \forall y_2 \exists X_1, X_2\\
&(\neg y_1 \rightarrow (S_1 \leftrightarrow S_I \wedge S_2 \leftrightarrow S_{IG})) \wedge (y_1 \rightarrow (S_1 \leftrightarrow S_{IG} \wedge S_2 \leftrightarrow S_G)) \wedge\\
&(\neg y_2 \rightarrow (X_1 \leftrightarrow S_1 \wedge X_2 \leftrightarrow S_{12})) \wedge (y_2 \rightarrow (X_1 \leftrightarrow S_{12} \wedge X_2 \leftrightarrow S_2)) \wedge\\
&I(S_I) \wedge G(S_G) \wedge T(X_1, X_2)
\end{aligned}$$



Compact Tree Encoding

- Log copies of transition relation
- $\log(k) + 1$ quantifier alternations

[Cashmore et al, 2012]

$$\begin{aligned}
&\exists S_I, S_G\\
&\exists X_2 \forall y_2\\
&\exists X_1 \forall y_1 \exists X_0\\
&((\neg y_1 \wedge \neg y_2) \rightarrow T(S_I, X_0)) \wedge ((y_1 \wedge y_2) \rightarrow T(X_0, S_G)) \wedge\\
&(\neg y_1 \rightarrow T(X_0, X_1)) \wedge (y_1 \rightarrow T(X_1, X_0)) \wedge\\
&((\neg y_2 \wedge y_1) \rightarrow T(X_0, X_2)) \wedge ((y_2 \wedge \neg y_1) \rightarrow T(X_2, X_0)) \wedge\\
&I(S_I) \wedge G(S_G)
\end{aligned}$$

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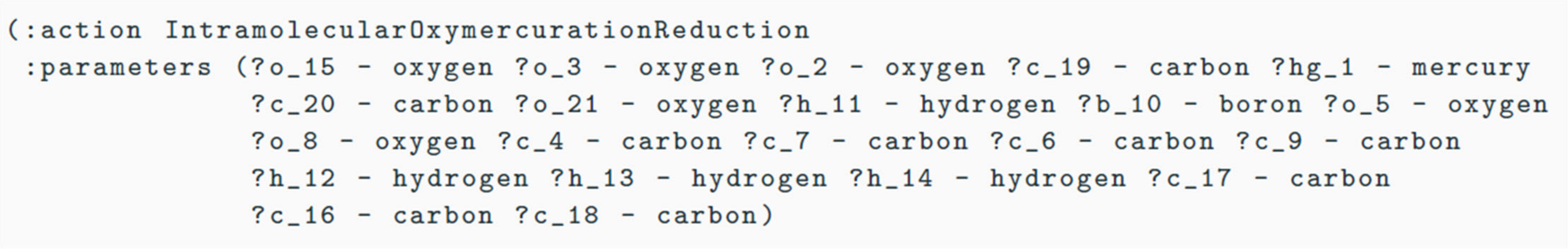
LIFTED CLASSICAL PLANNING AS QBF (1)

- Irfansha Shaik and Jaco van de Pol, *Classical planning as QBF without grounding*. In: ICAPS 2022 (International Conference on Automated Planning and Scheduling)
- **Grounding**: Instantiate actions for all **object combinations**

  apply_cnot(l1, l2, p1, p2, g1, g2, g3)

  $\mathbf{4 \times 4 \times 5 \times 5 \times 10 \times 10 \times 10 \;= 400,000}$ **potential instances!**

- **Main motivating example: Organic Synthesis (MIT exams organic chemistry)** [Masoumi et al, 2015], outstanding domain submission award IPC-2018



```
(:action IntramolecularOxymercurationReduction
 :parameters (?o_15 - oxygen ?o_3 - oxygen ?o_2 - oxygen ?c_19 - carbon ?hg_1 - mercury
             ?c_20 - carbon ?o_21 - oxygen ?h_11 - hydrogen ?b_10 - boron ?o_5 - oxygen
             ?o_8 - oxygen ?c_4 - carbon ?c_7 - carbon ?c_6 - carbon ?c_9 - carbon
             ?h_12 - hydrogen ?h_13 - hydrogen ?h_14 - hydrogen ?c_17 - carbon
             ?c_16 - carbon ?c_18 - carbon)
```

- **Note: in this case, the direct SAT encoding cannot even be generated!**

LIFTED CLASSICAL PLANNING AS QBF (2)

- We use Universal Quantifiers to enumerate all object combinations symbolically
- Result: QBF encoding remains small, even for organic synthesis
- QBF solver CAQE can handle this encoding reasonably well
  - Beats SAT-based planning by a large margin (Madagascar)
  - Competitive with the winning planning tools from IPC 2018 (FDSS, PowerLifted)
- We are only competitive if “grounding” is the bottleneck

$\exists \mathrm{A}^0, \mathrm{PM}^0, \ldots, \mathrm{A}^{k-1}, \mathrm{PM}^{k-1}$ …. *There exist Actions and Parameters*

$\forall \mathrm{OC}$ …. *Such that for all object combinations*

$\exists \mathrm{P}^0, \ldots, \mathrm{P}^k$ …. *There exists a plan of k steps*

$$\mathrm{I_u}(\mathrm{P}^0, \mathrm{OC}) \wedge \mathrm{G_u}(\mathrm{P}^k, \mathrm{OC}) \wedge \bigwedge_{i=0}^{k-1} \mathrm{T}_\mathrm{u}^i(\mathrm{P}^i, \mathrm{P}^{i+1}, \mathrm{OC}, \mathrm{A}^i, \mathrm{PM}^i)$$

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    - Tic-tac-toe, Hex, Breakthrough, Domineering

5. Validation of encodings with QBF certificates

ENCODING 2-PLAYER GAMES IN QBF

We used QBF to encode planning problems

QBF allows arbitrary quantifier alternation

Then why not encode 2-player games?

Player 1 can win the game in 3 moves:

$$\boldsymbol{\exists m_1 \forall m_2 \exists m_3 \forall m_4 \exists m_5} : \exists S_0, S_1, S_2, S_3, S_4, S_5 : \\ start(S_0) \land Move(S_0, m_1, S_1) \land \cdots \land Move(S_4, m_5, S_5) \land Won(S_5)$$

Needed:

- Domain-specific language to specify grid-based board games: BDDL
- Concise encoding from BDDL into QBF “there exists a winning strategy in d moves”

BDDL – BOARD-GAME DESCRIPTION

Irfansha Shaik, Jaco van de Pol, *Concise QBF Encodings for Games on a Grid.*
In: arXiv :2303.16949, 2023

Domain-specific modeling language for Grid-based Board Games

- Assume an $m \times n$ board of positions, and 2 players (Black, White)
- Describe **Black and White moves** for symbolic position (x, y)
  - Can refer to neighbours, like "*open*$(x, y+1)$" or "*white*$(x-2, y+1)$"
  - Implicitly observe the board boundaries
- Describe **Black and White winning conditions**
  - Patterns starting at symbolic position (x, y)
  - Example: black(x, y), black$(x, y+1)$, black$(x, y+2)$

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   - **Tic-tac-toe, Connect4, Breakthrough, Domineering, Hex**
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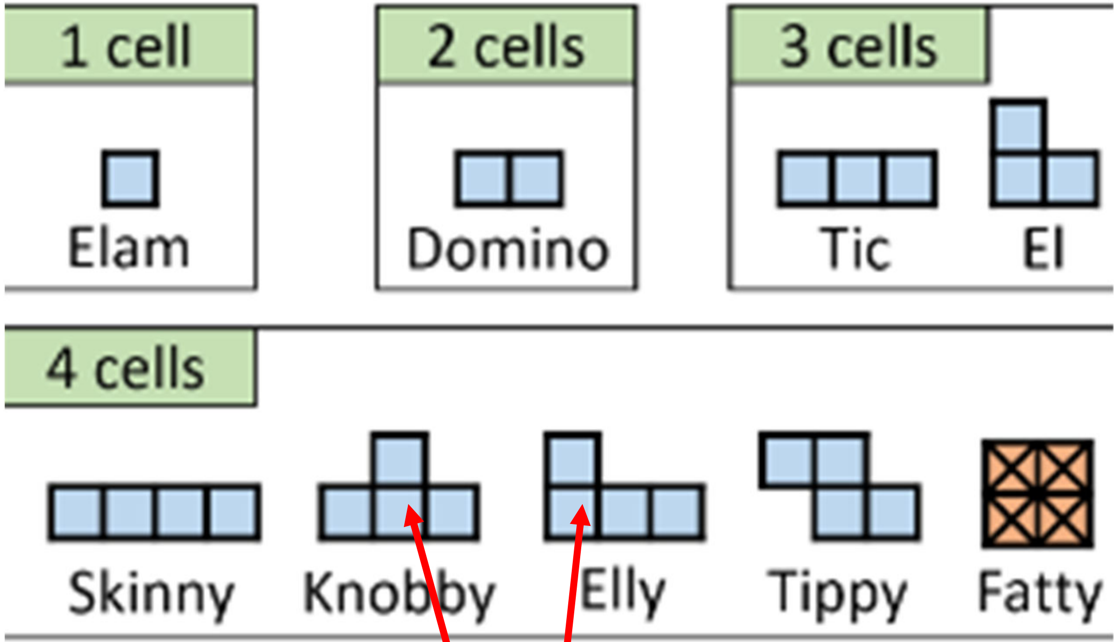
TIC-TAC-TOE (POSITIONAL GAME)

```
#blackactions
:action occupy
:parameters (?x,?y)
:precondition (open(?x,?y))
:effect (black(?x,?y))

#whiteactions
:action occupy
:parameters (?x,?y)
:precondition (open(?x,?y))
:effect (white(?x,?y))
```

GENERALIZED TTT (POSITIONAL GAME)

```
#blackgoals % Elly
( black(?x,?y) black(?x,?y+1)
  black(?x+1,?y) black(?x+2,?y))
( black(?x,?y) black(?x,?y+1)
  black(?x+1,?y) black(?x+1,?y))
...

#whitegoals % Knobby
( white(?x,?y) white(?x,?y+1)
  white(?x-1,?y) white(?x+1,?y)
...
```



[Diptamara et al, QBF @ SAT 2016]

(x,y)

CONNECT4 (NON-POSITIONAL GAME)

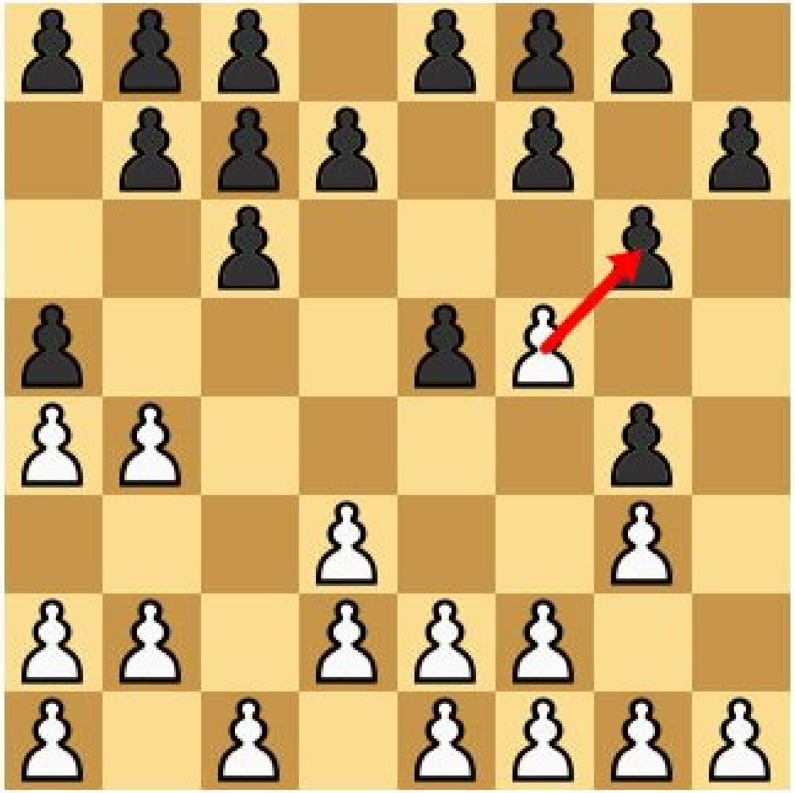
Move on (x, y) depends on other positions

```
#blackactions
:action occupy_on_top
:parameters (?x,?y)
:precondition
   (open(?x,?y) (not(open(?x,?y-1)))
:effect (black(?x,?y))

:action occupy_on_bottom
:parameters (?x,?y)
:precondition (open(?x, ymin))
:effect (black(?x, ymin))
```

BREAK-THROUGH (CAN TAKE PIECES)

"Chess with pawns"

```
#whiteactions
:action take_right
:parameters (?x,?y)
:precondition
  (white(?x,?y) black(?x+1,?y+1))
:effect
  (open(?x,?y) white(?x+1,?y+1))

#whitegoals
  (white(?x, ymax))
```

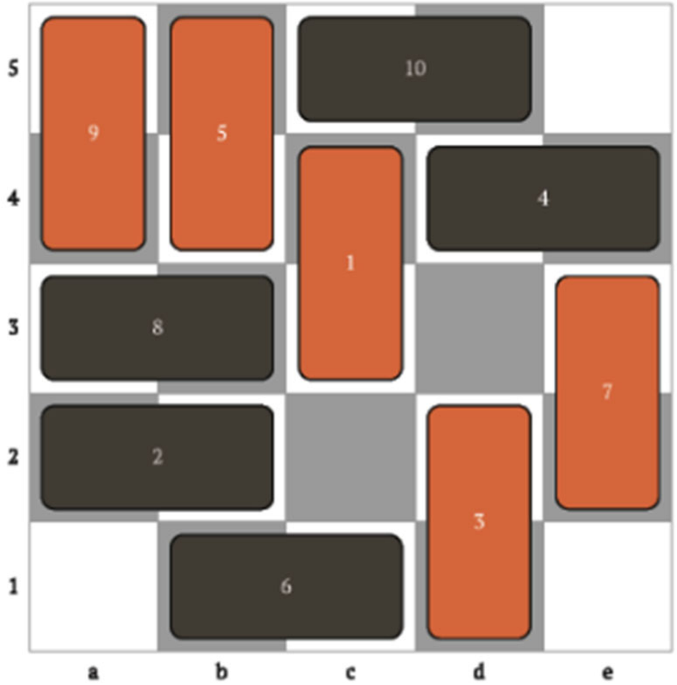


[Stephenson et al., preprint

DOMINEERING (MULTIPLE POSITIONS)

```
#blackactions
:action domino-horizontal
:parameters (?x,?y)
:precondition (open(?x,?y) open(?x+1,?y))
:effect (black(?x,?y) black(?x+1,?y))

#blackgoals
  False
```

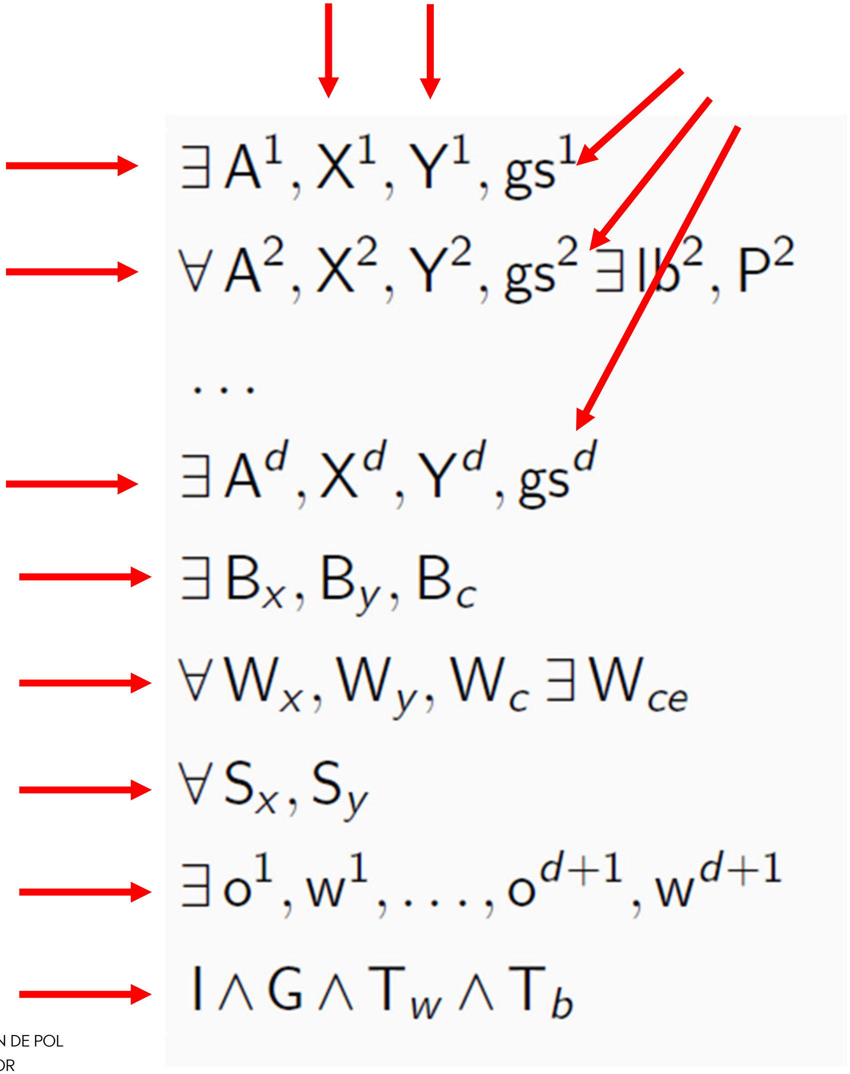
The first player that has no move loses the game



[J. Davies et al., Institute of Mathematics]

LIFTED QBF TRANSLATION: WIN IN $\leq d$ STEPS

- We use quantifier alternations to alternate between moves of the players.
- We encode positions using bitvectors. We introduce “Boolean circuitry” to handle “$?x \pm c$” and “$y_{min} \leq ?y \leq y_{max}$”
- We use “game-stop” variables to check (non)-winning of intermediate positions
- We use bitvectors for Black Winning and White Non-Winning conditions
- We use symbolic position variables to check for all positions symbolically
- We check that the resulting play is a valid winning play, for pos (S_x, S_y)



$$\exists \mathrm{A}^1, \mathrm{X}^1, \mathrm{Y}^1, \mathrm{gs}^1$$
$$\forall \mathrm{A}^2, \mathrm{X}^2, \mathrm{Y}^2, \mathrm{gs}^2 \, \exists \mathrm{lb}^2, \mathrm{P}^2$$
$$\ldots$$
$$\exists \mathrm{A}^d, \mathrm{X}^d, \mathrm{Y}^d, \mathrm{gs}^d$$
$$\exists \mathrm{B}_x, \mathrm{B}_y, \mathrm{B}_c$$
$$\forall \mathrm{W}_x, \mathrm{W}_y, \mathrm{W}_c \, \exists \mathrm{W}_{ce}$$
$$\forall \mathrm{S}_x, \mathrm{S}_y$$
$$\exists \mathrm{o}^1, \mathrm{w}^1, \ldots, \mathrm{o}^{d+1}, \mathrm{w}^{d+1}$$
$$\mathrm{I} \wedge \mathrm{G} \wedge \mathrm{T}_w \wedge \mathrm{T}_b$$

linear in input size of game and d,
$d + 3$ quantifier alternations

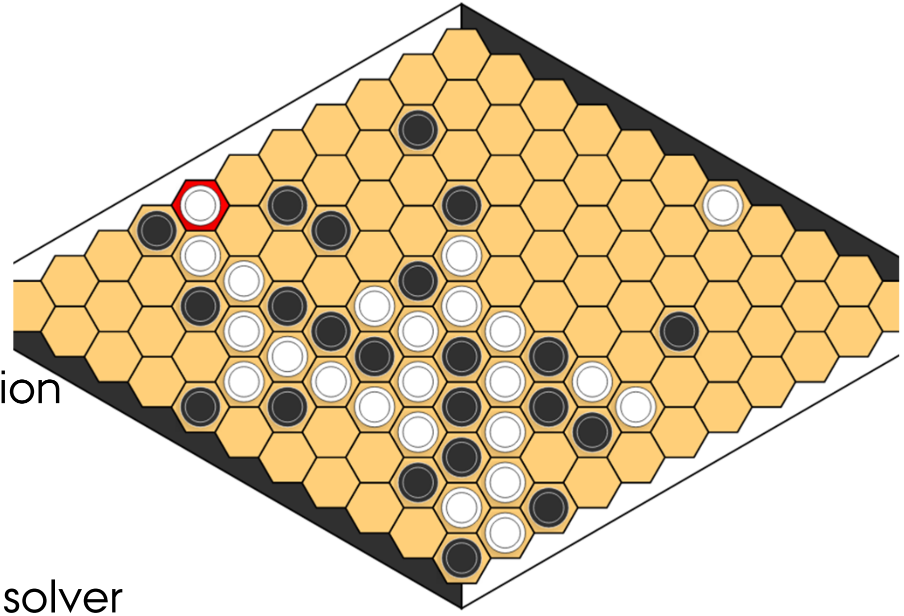
HEX (PIET HEIN)

Irfansha Shaik, V. Mayer-Eichberger, J vd Pol, A. Saffidine,

Implicit State and Goals in QBF Encodings for Positional Games.
In: arXiv 2301.07345, 2023

Hex: Back to positional games

- *But exponentially many winning patterns.*



Solution:

- Lifted encoding of winning path, using $N(x,y)$-relation
- Several encodings: (non)-lifted, (non)-CNF, …

Results:

- Can solve Hein's puzzles on small boards with QBF-solver
- Can encode and solve human-played end-games on large boards

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VALIDATION OF ENCODINGS IN QBF

Irfansha Shaik, Maximilian Heisinger, Martina Seidl, and Jaco van de Pol,
Validation of QBF Encodings with Winning Strategies. In: SAT 2023

Encoding in QBF is difficult: low-level, error-prone, resulting strategy is hard to test

Can we use certificates, which are already produced by QBF solvers?

- Validation of QBF solvers
- Validation of QBF encoding

Ideas:

- Can use the certificate to extract winning strategy ➔ interactive play
- Also: basis for automatic testing of invariants, equivalences, etc.

Scalability:

- **Full certificate:** checking is efficient, but certificates are enormous
- **Only witness of first layer:** certificates are small, but need many QBF queries
- **Partial certificates:** certificates are reasonable, checking is efficient

CONCLUSION

- We can solve interesting problems as classical planning problems
  - Optimal Quantum Layout Synthesis
  - Organic Synthesis
- QBF and QBF solvers can be used, based on concise encodings
  - Path compression
  - Lifted encodings
- The QBF encoding can be extended to 2-player games
  - Board games, symbolic positions
  - Hex, symbolic winning conditions
- Validation of encodings
  - Winning strategies can be validated, based on QBF certificates
  - Interactive play, invariant testing, equivalence testing

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