Miniworkshop on Geometry and Mechanics (Antwerp, May 30–31 2022)

- **Sándor Hajdú**: *Jacobi fields, conjugate points and nonlinear splittings in Finsler geometry and related fields.*

  We provide a method to find conjugate points for systems of second-order ordinary differential equations (SODEs) and we show that for a certain class, called sprays, a certain freedom in the choice of a projective factor can be exploited in the search for their conjugate points. Moreover, we investigate nonlinear splittings on fibre bundles. These can be thought of as generalizations of Ehresmann connections. We apply our results to the contexts of Finsler geometry and Lagrangian mechanics.

- **Marta Farré Puiggalí**: *Forced mechanical systems preserving first integrals.*

  We will discuss smooth and discrete dissipative systems which preserve constants of the motion induced by a Lie group of symmetries. We are interested in particular in preservation of angular momentum and in the construction of geometric integrators which preserve the constants of motion induced by the Lie group of symmetries.

- **David Martín de Diego**: *Retraction maps and symplectic integration.*

  In this first part of the talk, we will use a typical tool in nonlinear optimization (retraction maps) to derive symplectic integrators for Hamiltonian systems. We will see that is necessary to use some tools from geometric mechanics to derive their construction and properties (Lagrangian submanifolds, Tulzcyjew triple, complete lifts...).

  Moreover, we will explore how to extend this construction to Lagrangian systems with internal constraints (imposed by the singularity of a Lagrangian function), or external constraints determined by holonomic constraints.

- **Zoltan Muzsnay**: *Almost all Finsler metrics have infinite-dimensional holonomy group.*

  The holonomy group of a Riemann or Finsler manifold is the group generated by parallel translations along loops. It is known that the Riemannian holonomy group is a closed subgroup of the orthogonal group; in particular, it is always finite-dimensional. In the Finslerian case, the situation is different: there are classes of Finsler metrics where the holonomy group is finite-dimensional, but there are also some explicit examples where the holonomy group is infinite-dimensional. A natural and fundamental question is whether for a generic Finsler manifold the holonomy group is infinite-dimensional. We prove that for a generic Finsler manifold the holonomy group is infinite-dimensional and the set of Finsler metrics on a manifold contains an open, and everywhere dense subset of Finsler metrics with infinite-dimensional holonomy groups.

  The talk is based on joint work with V. Matveev (Friedrich Schiller University, Jena) and B. Hubicska (University of Debrecen).

- **Geoff Prince**: *Covariant derivatives for Ehresmann connections.*

  Covariant derivatives are constructed for some quite general Ehresmann connections on fibre bundles. A vertical endomorphism allows construction of covariant derivatives separately on both the vertical and horizontal distributions of the connection which can then be glued together on the total space. Application is made to some familiar situations.

- **David Saunders**: *Linearizing connections – the jet approach.*

  Given an Ehresmann connection on a bundle, we often wish to find a related linear connection on a vector bundle. We show how such a linearization procedure can be understood by regarding the connection as a section of a jet bundle, and we give three different constructions for a related linear connection. All three constructions can be used when the original bundle is an affine bundle, but we shall see that this constraint can be relaxed when we use the third construction: in that case the fibres of the bundle need not be affine spaces, provided that each fibre has a parallel translation given by an auxiliary covariant derivative."

- **Marco Zambon**: *Multisymplectic observables and higher Courant algebroids.*

  I will report on joint work with Antonio Miti. Consider a closed, non-degenerate differential form ω of any degree. Associated to it there is an L_∞-algebra of observables, and an L_∞-algebra of sections of the higher Courant algebroid twisted by ω. Our main result is that there is an L_∞-embedding of former into the latter. We display explicit formulae for the embedding, involving the Bernoulli numbers. For symplectic forms this is reduces to a prequantization map, and when ω is a 3-form the embedding was found by Rogers around 2012.